Data analysis for LISA extreme mass ratio capture sources

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- Desire to detect many EMRI’s is driving the specification for the floor of the LISA noise curve.
Example waveform

![Example waveform](image_url)
Detection of EMRI’s

- The parameter space is very large, waveforms depend on 14 different parameters - \((M, S, m, e, r_p, \iota, \psi_0, \chi_0, \phi_0, \theta_K, \phi_K, \theta_s, \phi_s, D)\).
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- Scoping out data analysis using kludged inspiral waveforms, as more accurate waveforms are presently unavailable.
Data analysis strategy

- Use Buonnano, Chen, Vallisneri trick to search 5 extrinsic parameters automatically. Use FFT to search time offset cheaply.
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- Estimate optimal SNR required for detection by this method as \(SNR_{\text{thresh}} \sim 34\) for pessimistic case (3yrs/2wks), and \(SNR_{\text{thresh}} \sim 36\) for optimistic case (5yrs/3wks). Compare this to optimal SNR’s computed using synthetic LISA.
Astrophysical event rates

- Use galaxy luminosity function and \( L - \sigma / M - \sigma \) relations to estimate space density of black holes

\[
M_\bullet \frac{dN}{dM_\bullet} = 1.5 \times 10^{-3} h_{65}^2 \text{ Mpc}^{-3}.
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(1)

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- Use capture rates from Freitag’s Milky Way simulation. Scale these to other galaxies by assuming an $M^{3/8}$ dependence.

- Conservative rates could be a factor of $\sim 100$ smaller for WDs, or a factor of $\sim 10$ smaller for black holes.
<table>
<thead>
<tr>
<th>$M_\odot$</th>
<th>space density $10^{-3} h_{65}^2 \text{Mpc}^{-3}$</th>
<th>Merger rate $\mathcal{R}$ Gpc$^{-3} y^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_\odot$</td>
<td>$0.6 M_\odot$ WD</td>
<td>$1.4 M_\odot$ MWD/NS</td>
</tr>
<tr>
<td>$10^{6.5\pm0.25}$</td>
<td>1.7</td>
<td>8.5</td>
</tr>
<tr>
<td>$10^{6.0\pm0.25}$</td>
<td>1.7</td>
<td>6</td>
</tr>
<tr>
<td>$10^{5.5\pm0.25}$</td>
<td>1.7</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Table II: Merger rates
LISA detection rates

- Put this together to estimate expected number of detections. Consider four cases - optimistic/pessimistic and LISA/‘short LISA’. For $z > 1$, system evolution is uncertain and flat space extrapolation is no longer valid, so we quote $z < 1$ lower limits (*).

<table>
<thead>
<tr>
<th>$M_\bullet$</th>
<th>$m$</th>
<th>LISA</th>
<th>Short LISA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Optimistic</td>
<td>Pessimistic</td>
</tr>
<tr>
<td>300 000</td>
<td>0.6</td>
<td>8</td>
<td>0.7</td>
</tr>
<tr>
<td>300 000</td>
<td>10</td>
<td>739</td>
<td>89</td>
</tr>
<tr>
<td>300 000</td>
<td>100</td>
<td>1*</td>
<td>1*</td>
</tr>
<tr>
<td>1 000 000</td>
<td>0.6</td>
<td>94</td>
<td>9</td>
</tr>
<tr>
<td>1 000 000</td>
<td>10</td>
<td>1000*</td>
<td>800</td>
</tr>
<tr>
<td>1 000 000</td>
<td>100</td>
<td>1*</td>
<td>1*</td>
</tr>
<tr>
<td>3 000 000</td>
<td>0.6</td>
<td>67</td>
<td>2</td>
</tr>
<tr>
<td>3 000 000</td>
<td>10</td>
<td>1700*</td>
<td>134</td>
</tr>
<tr>
<td>3 000 000</td>
<td>100</td>
<td>2*</td>
<td>1*</td>
</tr>
</tbody>
</table>
Summary

• Preliminary results are very promising - suggest we should detect $\sim 10^3$ EMRI’s during LISA’s lifetime.

• BH rates are robust to more conservative assumptions, although WDs become marginal.

• Remaining issues -
  ★ Firm up template counts, and optimize division of computational resources.
  ★ Comparison to accurate Teukolsky and self-force waveforms.
  ★ Effect of self-confusion on data analysis.
  ★ Improve estimates of capture rates and orbital parameter distributions.