Constraining population synthesis (and binary black hole inspiral rates) using binary neutron stars

Richard O’Shaughnessy

GWDAW-8

12-17-2003
Outline

• Background and Motivation: Population synthesis
  – Fundamental approach to rates… but poor constraints (e.g. BBH merger rate)
  – New idea: Use neutron star rate as guide…

• It is possible!

• Constrain models for binary evolution

• Constrain BBH merger rate
  – Computational issues

• Results ➔ Present status and future plans
Population Synthesis

Models

Probabilities, rates, ...

Many parameters…

but

1) many have **narrow** ranges
2) dependence is smooth
3) …probably correlations too

\( n+1 \) dimensions

Scale factor
number of stars
... Properties at formation
metallicity of gas
initial mass distribution
...
Models for binary evolution
supernova kick magnitude
common envelope efficiency
...
Population Synthesis: Results

Despite best constraints on models

→ broad range of compact object merger rates:

Example:

Supernova kick magnitude
Idea:
NS Rate $\rightarrow$ constraints

Empirical distribution of binary NS merger rate $\rightarrow$

1. Constraints on population synthesis models

2. Distribution of BBH merger rates

(assuming equal prior probabilities of models)
Computational issues I: Computation time

Computation time

- **Number of models we can evaluate:**
  \[
  (1 \text{ model/20 minutes/node}) \times (1 \text{ year}) \times 10^3 \text{ nodes} \\
  \approx 10^7 \text{ models}
  \]

- **Intensive naïve approach:**
  - 11 dimensional space
  - 10 points per dimension
  \[
  \Rightarrow 10^{11} \text{ models}
  \]
  \[
  \Rightarrow 10^4 \text{ years (!)}
  \]

... **but** some parameters have narrow ranges

- **Revised naïve approach:**
  - 10 points needed in each of 3 dimensions
  - 3 points needed in others
  \[
  \Rightarrow 10^7 \text{ models}
  \]

This is search by exhaustion
  \[
  \Rightarrow \text{can do even better...}
  \]
Application I: Constrain Model Parameters

Binary NS merger rate: 95% confidence interval

Generally: For any merger rate, n-1 dimensional manifold of model parameters consistent with rate

Application: Find confidence-interval boundaries in model space
Computational issues II: Root finding via genetic algorithms

• **Need**: Robust way to **find** n-1 dimensional manifold of **all** solutions

• **Solution**: Genetic algorithms
  (still under development)

  *Example*: Finding a single root
  – **Robust**: noisy functions, multiple maxima, high dimensions
  – **Efficient**: Exponential convergence
Application II: BBH merger rate distribution

Monte carlo:

1. Select many random models $m_k$ (=equal prior probability)
2. Count number of models with BH merger rates in a bin B, weighted by binary NS rate:

$$\text{count}(B) = \sum_k \Theta(r_b(m_k) \in B) \frac{p_n(r_n(m_k))}{\text{const}}$$

$\Rightarrow$ histogram
Probability distribution formulae

• Explicit formula:

\[ A_n(m) = \int d\bar{m} \delta(r_n(\bar{m}) - r_n(m)) \]

\[ p_m(m) = \frac{p_n(r_n(m))}{A_n(m)} \]

\[ p_b(R_b) = \int d\bar{m} p_m(\bar{m}) \delta(R_b - r_b(\bar{m})) \]

Key

- \( r_n(m) \): rate of binary NS merger for model \( m \)
- \( r_b(m) \)
- \( p_n(r)dr \): probability for binary NS rate to be in \([r, r+dr]\)
Status

• Present status:
  – Constrain population synthesis
    • Algorithm development
  – Determine BH merger rate distribution
    • Monte Carlo accumulating…
  – Also…looking for simplifications (correlations, etc)

• Future plans:
  – Improvements
    • Include for prior probability distributions for parameters (e.g. supernova kick distribution)
    • Include more constraints (e.g. statistics of x-ray binaries; etc.)
  – Additions
    • Account for other known uncertainties in analysis (e.g. statistical fluctuations in rate calculations)
  – Long-term improvements (exploring):
    • Nonrandom (bayesian) searches [=maximize information obtained at each step]