Network analysis for coalescing binaries: 
coherent vs coincidence based strategies

Andrea Viceré

19th December 2003

Università degli Studi di Urbino
Motivation and context

✔ It is well known that in gaussian noise a coherent network search of CB events is optimal

Motivation and context

✔ It is well known that in gaussian noise a *coherent* network search of CB events is optimal


✔ The resulting computational cost is however high: $O(\text{TFlops})$ for networks comprising more than 3 detectors

Motivation and context

✓ It is well known that in gaussian noise a *coherent* network search of CB events is optimal


✓ The resulting computational cost is however high: $O$(TFlops) for networks comprising more than 3 detectors


✓ How does the coherent search compare with OR-based and AND-based strategies? Are there compromise solutions?

✓ I considered the case of NS-NS binaries for simplicity, and parameters of the existing network of ITFs
The global network

Left: network from above US  Right: from above EU
The global network

Left: network from above US    Right: from above EU

Black lines represent the ITF axes.
**The global network**

**Left:** network from above US    **Right:** from above EU

**Black** lines represent the ITF axes.

**Colored** lines are the axes of the detector and Earth frames: **Z** crosses the North pole, **X** crosses the Greenwich meridian.
Design sensitivities of the individual detectors

They were used to estimate the sensitivity scale to NS-NS binaries

\[ \text{sens} \propto \sqrt{\int \frac{f^{-7/3}}{S_n(f)} df}. \]
The averaged response of the global network

✓ The network response depends on the source direction $\theta, \varphi$, the binary inclination $\varepsilon$ and the wave polarization $\psi$. 
The averaged response of the global network

✔ The network response depends on the source direction $\theta, \phi$, the binary inclination $\varepsilon$ and the wave polarization $\psi$.

✔ Averaging over $\varepsilon$ and $\psi$ one can plot the average cumulative SNR available to the network as a whole, as a function of the direction in the sky.
Individual contributions to the network SNR
Individual contributions to the network SNR
Individual contributions to the network SNR

Left: LIGO network; center: GEO and Virgo network; right: TAMA
Individual contributions to the network SNR

**Left:** LIGO network; **center:** GEO and Virgo network; **right:** TAMA

✔ Note the different Virgo-GEO antenna pattern, which contributes to a more spherical overall pattern.
Rules for the comparison

✔ Set a false alarm rate of the network as a whole (1 event/year)
Rules for the comparison

✔ Set a false alarm rate of the network as a whole (1 event/year)

✔ Generate events with random direction $\theta, \phi$ and source parameters $\epsilon, \psi$, but the same network SNR. This means turning the response peanut into a sphere, to compare the strategies in a way independent from the source direction/polarization.
Rules for the comparison

✔ Set a false alarm rate of the network as a whole (1 event/year)

✔ Generate events with random direction $\theta, \phi$ and source parameters $\varepsilon, \psi$, but the same network SNR. This means turning the response peanut into a sphere, to compare the strategies in a way independent from the source direction/polarization.

✔ Set false alarm rates $R_{FA}$ on the individual detectors, and rules to combine the events that lead to the same overall $R_{FA}$ as the “coherent network”.
Rules for the comparison

✔ Set a false alarm rate of the network as a whole (1 event/year)

✔ Generate events with random direction $\theta, \varphi$ and source parameters $\varepsilon, \psi$, but the same network SNR. This means turning the response peanut into a sphere, to compare the strategies in a way independent from the source direction/polarization.

✔ Set false alarm rates $R_{FA}$ on the individual detectors, and rules to combine the events that lead to the same overall $R_{FA}$ as the “coherent network”.

✔ Compute the SNR seen by each detector, hence local detection probabilities $P_{DET}$ for each sampled direction/polarization.
Rules for the comparison

✔ Set a false alarm rate of the network as a whole (1 event/year)

✔ Generate events with random direction $\vartheta, \varphi$ and source parameters $\epsilon, \psi$, but the same network SNR. This means turning the response peanut into a sphere, to compare the strategies in a way independent from the source direction/polarization.

✔ Set false alarm rates $R_{FA}$ on the individual detectors, and rules to combine the events that lead to the same overall $R_{FA}$ as the “coherent network”.

✔ Compute the SNR seen by each detector, hence local detection probabilities $P_{DET}$ for each sampled direction/polarization.

✔ Combine with various strategies (OR, AND); obtain the average $P_{DET}$
Rules for the comparison

✔ Set a false alarm rate of the network as a whole (1 event/year)

✔ Generate events with random direction $\theta, \phi$ and source parameters $\epsilon, \psi$, but the same network SNR. This means turning the response peanut into a sphere, to compare the strategies in a way independent from the source direction/polarization.

✔ Set false alarm rates $R_{FA}$ on the individual detectors, and rules to combine the events that lead to the same overall $R_{FA}$ as the “coherent network”.

✔ Compute the SNR seen by each detector, hence local detection probabilities $P_{DET}$ for each sampled direction/polarization.

✔ Combine with various strategies (OR, AND); obtain the average $P_{DET}$

✔ Compare with the coherent case, and vary the SNR available to the network.
Statistics

It is worth recalling that the $\text{SNR}^2$ seen by the individual detectors and by the network obey to different statistics.
Statistics

It is worth recalling that the $\text{SNR}^2$ seen by the individual detectors and by the network obey to different statistics

✔️ On a single detector the $\text{SNR}^2$ is a $\chi^2$ with 2 DOF, hence if $\xi$ is a threshold

\[
P_{\text{FA}}(\xi) = e^{-\xi} \; ; \; P_{\text{DET}}(\xi, E_{\text{sig}}) = \int_{\xi}^{\infty} e^{-E-E_{\text{sig}}} I_0 \left(2 \sqrt{E \ast E_{\text{sig}}} \right) dE
\]
Statistics

It is worth recalling that the $\text{SNR}^2$ seen by the individual detectors and by the network obey to different statistics

✔ On a single detector the $\text{SNR}^2$ is a $\chi^2$ with 2 DOF, hence if $\xi$ is a threshold

$$P_{FA}(\xi) = e^{-\xi}; \quad P_{DET}(\xi, E_{\text{sig}}) = \int_{\xi}^{\infty} e^{-E-E_{\text{sig}}} I_0(2\sqrt{E*E_{\text{sig}}}) \, dE$$

✔ On the network, the corresponding quantity is a $\chi^2$ with 4 DOF, hence

$$P_{FA}(\xi) = (1 + \xi) e^{-\xi}; \quad P_{DET}(\xi, E_{\text{sig}}) = \int_{\xi}^{\infty} \sqrt{\frac{E}{E_{\text{sig}}}} e^{-E-E_{\text{sig}}} I_1(2\sqrt{E*E_{\text{sig}}}) \, dE$$
Statistics

It is worth recalling that the $\text{SNR}^2$ seen by the individual detectors and by the network obey to different statistics.

✔️ On a single detector the $\text{SNR}^2$ is a $\chi^2$ with 2 DOF, hence if $\xi$ is a threshold

$$P_{\text{FA}}(\xi) = e^{-\xi}; \quad P_{\text{DET}}(\xi, E_{\text{sig}}) = \int_{\xi}^{\infty} e^{-E-E_{\text{sig}}} I_0(2\sqrt{E \ast E_{\text{sig}}}) \, dE$$

✔️ On the network, the corresponding quantity is a $\chi^2$ with 4 DOF, hence

$$P_{\text{FA}}(\xi) = (1 + \xi) e^{-\xi}; \quad P_{\text{DET}}(\xi, E_{\text{sig}}) = \int_{\xi}^{\infty} \sqrt{\frac{E}{E_{\text{sig}}}} e^{-E-E_{\text{sig}}} I_1(2\sqrt{E \ast E_{\text{sig}}}) \, dE$$

This is just to remind that the interpretation of the $\text{SNR}$ clearly depends on the kind of statistic, and we have to refer to $P_{\text{DET}}, P_{\text{FA}}$ for a meaningful comparison.
Coherent vs OR network

Errors represent the RMS spread due to the non-uniform antenna patterns.
Coherent vs OR network

✔ Errors represent the RMS spread due to the non-uniform antenna patterns.

✔ Dotted lines are the “best” and “worst” response of the OR network.
Errors represent the RMS spread due to the non-uniform antenna patterns.

Dotted lines are the “best” and “worst” response of the OR network.

The “best” attains the coherent result because of directions along which only one detector responds.
Assume an higher local $R_{FA} \Rightarrow P_{DET}$ close to the coherent case.
Coherent vs OR with higher FA rate

✔ Assume an higher local $R_{FA} \Rightarrow P_{DET}$ close to the coherent case.

✔ **Unfair:** the $R_{FA}$ of this “OR” network is way larger than for a coherent one.
Coherent vs OR with higher FA rate

✔ Assume an higher local $R_{FA} \Rightarrow P_{DET}$ close to the coherent case.

✔ **Unfair:** the $R_{FA}$ of this “OR” network is way larger than for a coherent one.

✔ **But,** as a pre-selection, does not kill events seen by a coherent follow-up.
Two detectors must flag the event. Tune local $R_{FA}$ for a fair comparison.
Coherent vs AND with 2 detectors

✔ Two detectors must flag the event. Tune local $R_{FA}$ for a fair comparison.

✔ In average, at larger SNR the AND(2) gets close to the coherent case.
Two detectors must flag the event. Tune local $R_{FA}$ for a fair comparison.

In average, at larger SNR the AND(2) gets close to the coherent case.

The minimum is always zero: there exist directions/polarizations that only one detector is sensitive to!
✔ Now at least three detectors need to be above the local threshold.
Coherent vs AND with 3 detectors

Now at least three detectors need to be above the local threshold.

The result is slightly worse, but not qualitatively different: there exist blind directions/polarizations.
Conclusions

This study has limitations: it is based just on amplitudes/polarizations, not on delays, so it should be taken as exploratory: a fuller Monte Carlo is needed.
Conclusions

This study has limitations: it is based just on amplitudes/polarizations, not on delays, so it should be taken as exploratory: a fuller Monte Carlo is needed.

Yet, it seems possible to conclude something:

✔ A fully coherent analysis can be made “economical”. We can perform it on events selected on the individual detectors.
Conclusions

This study has limitations: it is based just on amplitudes/polarizations, not on delays, so it should be taken as exploratory: a fuller Monte Carlo is needed.

Yet, it seems possible to conclude something:

✔ A fully coherent analysis can be made “economical”. We can perform it on events selected on the individual detectors.

✔ This method will slightly lower the detection probability, at SNR values that probably we would not trust anyway, because of non-gaussian noise tails.
Conclusions

This study has limitations: it is based just on amplitudes/polarizations, not on delays, so it should be taken as exploratory: a fuller Monte Carlo is needed.

Yet, it seems possible to conclude something:

✔ A fully coherent analysis can be made “economical”. We can perform it on events selected on the individual detectors.

✔ This method will slightly lower the detection probability, at SNR values that probably we would not trust anyway, because of non-gaussian noise tails.

✔ Operating detectors in AND to fight the non-gaussian noise we pay a price, because not every direction is well covered by at least two detectors.
Conclusions

This study has limitations: it is based just on amplitudes/polarizations, not on delays, so it should be taken as exploratory: a fuller Monte Carlo is needed.

Yet, it seems possible to conclude something:

✔ A fully coherent analysis can be made “economical”. We can perform it on events selected on the individual detectors.

✔ This method will slightly lower the detection probability, at SNR values that probably we would not trust anyway, because of non-gaussian noise tails.

✔ Operating detectors in AND to fight the non-gaussian noise we pay a price, because not every direction is well covered by at least two detectors.

✔ This should be considered when planning new large detectors!
Conclusions

This study has limitations: it is based just on amplitudes/polarizations, not on delays, so it should be taken as exploratory: a fuller Monte Carlo is needed.

Yet, it seems possible to conclude something:

✔ A fully coherent analysis can be made “economical”. We can perform it on events selected on the individual detectors.

✔ This method will slightly lower the detection probability, at SNR values that probably we would not trust anyway, because of non-gaussian noise tails.

✔ Operating detectors in AND to fight the non-gaussian noise we pay a price, because not every direction is well covered by at least two detectors.

✔ This should be considered when planning new large detectors!

The reduction of $P_{\text{DET}}$ due to the AND can be avoided only if we can fully trust the vetoes used on the individual detectors.