Coherent Coincident Analysis of LIGO Burst Candidates

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Post Coincidence Coherent Analysis

- Burst candidates separately identified in the data stream of each interferometer by the Event Trigger Generators (ETG): TFclusters, Excess Power, WaveBurst, BlockNormal.
  - Tuning maximizes detection efficiency for given classes of waveforms and a given false rate ~ 1-2 Hz

- Multi-interferometer coincidence analysis:
  - Rule of thumb: detection efficiency in coincidence ~ product of efficiency at the single interferometers. Coincidence selection criteria should not further reduce the detection efficiency. The final false rate limits how loose the cuts can be.
  - Currently implemented: time and frequency coincidence (in general, different tolerance for different trigger generators).
  - Amplitude/energy cut: not yet implemented.

- Cross-Correlation for coherent analysis of coincident events
  - This is a waveform consistency test.
  - Allows suppression of false events without reducing the detection efficiency of the pipeline.
r-statistic Cross Correlation Test

For each triple coincidence candidate event produced by the burst pipeline (start time, duration $\Delta T$) process pairs of interferometers:

- Data Conditioning:
  - 100-2048 Hz band-pass
  - Whitening with linear error predictor filters

Partition the trigger in sub-intervals (50% overlap) of duration $\tau =$ integration window (20, 50, 100 ms). For each sub-interval, time shift up to 10 ms and build an $r$- statistic series distribution.

$$ r_k = \frac{\sum_{i}(x_i - \bar{x})(y_{i+k} - \bar{y})}{\sqrt{\sum_{i}(x_i - \bar{x})^2 \sum_{i}(y_{i+k} - \bar{y})^2}} $$

If the distribution of the $r$-statistic is inconsistent with the no-correlation hypothesis: find the time shift yielding maximum correlation confidence $C_M(j)$ ($j =$ index for the sub-interval)

Max confidence: $C_M(\tau_0) = 13.2$ at lag $= -0.7$ ms
CM(j) plots

- Each point: max confidence $C_{M}(j)$ for an interval $\tau$ wide (here: $\tau = 20$ms)

- Threshold on $\Gamma$:
  - 2 interferometers:
    \[ \Gamma = \max_j(C_{M}(j)) > \beta_2 \]
  - 3 interferometers:
    \[ \Gamma = \frac{\max_j(C_{M}^{12} + C_{M}^{13} + C_{M}^{23})}{3} > \beta_3 \]

In general, we can have $\beta_2 \neq \beta_3$

$\beta_3 = 3$: 99.9% correlation probability in each sub-interval

Testing 3 integration windows:
20ms ($\Gamma_{20}$) 50ms ($\Gamma_{50}$) 100ms ($\Gamma_{100}$)
in OR: $\Gamma = \max(\Gamma_{20}, \Gamma_{50}, \Gamma_{100})$
Exploring the test performance for triple coincidence detection, independently from trigger generators and from previous portions of the analysis pipeline:

- Add simulated events to real noise at random times in the 3 LIGO interferometers, covering 10% of the S2 dataset (in LIGO jargon: triple coincidence playground)
- Apply r-statistic test to 200 ms around the simulation peak time

**Definition of quantities used to characterize a burst signal:**

\[
\begin{align*}
  h_{rss} &= \left( \int_0^\infty |h(t)|^2 dt \right)^{1/2} = \left( \int_0^\infty |\tilde{h}(f)|^2 df \right)^{1/2} \\
  \text{SNR} &= \left( 2 \int_0^\infty \frac{|\tilde{h}(f)|^2}{S_h(f)} df \right)^{1/2} \frac{h_{rss}}{\sqrt{S_h(f_c)}} \approx \frac{h_{rss}}{\sqrt{S_h(f_c)}}
\end{align*}
\]

Total energy in the burst (units: strain/\(\text{rtHz}\)) [directly comparable to sensitivity curves]

SNR definition for excess-power techniques in the burst search = \(\text{SNR}_{\text{matched filtering}} / \sqrt{2}\)

For narrow-band bursts with central frequency \(f_c\)

\(S_h(f)\)=single-sided reference noise in the S2 Science Run

⇒ reference S2 SNR for a given amplitude/waveform
Detection Efficiency for Narrow-Band Bursts

Sine-Gaussian waveform $f_0=254$ Hz $Q=9$
linear polarization, source at zenith

$$h(t) = h_{\text{peak}} \sin(2\pi f_0(t-t_0)) e^{-(t-t_0)^2/\tau^2} \quad Q=\sqrt{2\pi f_0}$$

SNR $> 30$

50% triple coincidence detection probability:

$$h_{\text{peak}} = 3.2e-20 \ [\text{strain}] \quad h_{\text{rss}} = 2.3e-21 \ [\text{strain/rtHz}]$$

SNR:

- LLO-4km=8
- LHO-4km=4
- LHO-2km=3

$\sqrt{2|h(f)|} \ [\text{strain/Hz}]$

SNR $= \left[ \frac{\int_0^{\infty} |h(f)|^2 df}{\sqrt{\int_0^{\infty} |\tilde{h}(f)|^2 S_h(f) df}} \right] \approx \frac{h_{\text{rss}}}{\sqrt{S_h(f_c)}}$

$h_{\text{rss}} = \sqrt{\frac{\int_0^{\infty} |h(t)|^2 dt}{\int_0^{\infty} |\tilde{h}(f)|^2 df}} = \frac{Q}{4\sqrt{\pi f_0}} h_{\text{peak}}$

$f_{\text{char}}, h_{\text{rss}} \ [\text{strain/rtHz}]$
with 50% triple coincidence detection probability

Single-sided noise spectrum $\sqrt{S_h(f)} [\text{strain/rtHz}]$

Single-sided $h(t) [\text{strain/Hz}]$
Detection Efficiency for Broad-Band Bursts

Gaussian waveform $\tau = 1\text{ms}$
linear polarization, source at zenith

$$h(t) = h_{\text{peak}} e^{-(t-t_0)^2/\tau^2}$$

$\text{SNR} > 30$

$\text{SNR: LLO-4km} = 11.5 \quad \text{LHO-4km} = 6 \quad \text{LHO-2km} = 5$

50% triple coincidence detection probability:

$h_{\text{peak}} = 1.6e-19 \text{ [strain]} \quad h_{\text{rss}} = 5.7e-21 \text{ [strain/rtHz]}$

$\text{SNR: LLO-4km} = 11.5 \quad \text{LHO-4km} = 6 \quad \text{LHO-2km} = 5$
Detection Probability versus False Alarm Probability.
Parameter: triple coincidence confidence threshold $\beta_3$

Simulated 1730 events at fixed $h_{\text{peak}}, h_{\text{rss}}$ (10 events uniformly distributed in each S2 “playground” segment)

Tested cross correlation over 200 ms around the peak time

Operating condition: $\beta_3 = 3$

chosen from first principles (99.9% correlation probability in each event sub-interval for a pair of interferometers), corresponds to a ~1% false alarm probability for triple coincidence events with duration 200 ms.
Suppression of Accidental Coincidences from the Pipeline

In general: depends on the Event Trigger Generator and the nature of its triggers.
In particular: typical distribution of event duration (larger events have more integration windows).
Shown here: TFCLUSTERS 130 - 400 Hz (presented in Sylvestre’s talk)

<table>
<thead>
<tr>
<th></th>
<th>Singles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>LLO-4km (L1)</td>
<td>2.5 Hz</td>
</tr>
<tr>
<td>LHO-4km (H1)</td>
<td>2 Hz</td>
</tr>
<tr>
<td>LHO-2km (H2)</td>
<td>2 Hz</td>
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Coincident numbers reported here are averages of 6 background measurements:
LLO-LHO = ± 8, ± 6, ± 4 sec (H1-H2 together)

“Loose” coincidence cuts

<table>
<thead>
<tr>
<th>coincidence</th>
<th>triple coincident clusters ($\Delta t = 30$ ms)</th>
<th>after frequency cut (200Hz tolerance)</th>
<th>after r-statistic test ($\beta_3 = 3$)</th>
<th>Rejection efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1-H1-H2</td>
<td>20 mHz</td>
<td>15 mHz</td>
<td>0.1 mHz</td>
<td>(99.35 ± 0.08)%</td>
</tr>
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“Tight” coincidence cuts

<table>
<thead>
<tr>
<th>coincidence</th>
<th>triple coincident clusters ($\Delta t = 15$ ms)</th>
<th>after frequency cut (75Hz tolerance)</th>
<th>after r-statistic test ($\beta_3 = 3$)</th>
<th>Rejection efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1-H1-H2</td>
<td>6 mHz</td>
<td>1 mHz</td>
<td>0.01 mHz (1/day)</td>
<td>(98.8 ± 0.4)%</td>
</tr>
</tbody>
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PRELIMINARY!!
In general: depends on the trigger generators and the previous portion of the analysis pipeline (typical event duration, how stringent are the selection and coincidence cuts)

Shown here: TFCLUSTERS 130-400 Hz with “loose” coincidence cuts
Conclusions

- The LIGO burst S1 analysis exclusively relied on event trigger generators and time/frequency coincidences.

- The search in the second science run (S2) includes a new module of coherent analysis, added at the end of the burst pipeline:
  - r-statistic test for cross correlation in time domain
    » Assigns a confidence to coincidence events at the end of the burst pipeline
    » Verifies the waveforms are consistent
    » Suppresses false rate in the burst analysis, allowing lower thresholds

- Tests of the method, using simulated signals on top of real noise, yield 50% triple coincidence detection efficiency for narrow-band and broad-band bursts at SNR=3-5 in the least sensitive detector (LHO-2km) with a false probability ~1%.

- Currently measuring global efficiency and false rate for the S2 pipeline (event analysis + coherent analysis).