Gravitational Wave Astronomy using 0.1Hz space laser interferometer

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In 2001 we considered what we can do using 0.1 hertz laser interferometer (Seto, Kawamura and TN: PRL 87 221103)
Motivation to DECIGO comes from extra solar planets

- Many extra solar planets are found using many absorption lines (~5000) of nearby G type stars since small orbital motion up to 10m/s can be measured.
- Loeb (1998) proposed to apply this techniques to many QSO absorption lines so that two observations between ten years yield direct measurement of Cosmic Acceleration and thus dark energy.

Our point is

- Use gravitational waves from coalescing binary neutron stars at z=1 instead of QSO absorption lines.
- Then the frequency of GW (a year to ten years before the coalescence) should be 0.1 Hz band where little proposal for detectors existed.
Our Point and Strategy

• Consider the ultimate possible detector in the spirit of

• *necessity is the mother of the invention*

• We call the detector DECIGO (DECi hertz laser Interferometer Gravitational wave Observatory)

• *We may not see the construction of DECIGO in our life since highly advanced technology is needed,* we are sure that our children or grandchildren will *decide and go DECIGO.*
\[ f = 0.1 \text{Hz} (1 + z)^{-1} \left( \frac{M_t}{2.8 M_\odot} \right)^{1/2} \left( \frac{a}{15500\text{km}} \right)^{-3/2} \]

\[ t_c = 7(1 + z) \left( \frac{M_1}{1.4 M_\odot} \right)^{-1} \left( \frac{M_2}{1.4 M_\odot} \right)^{-1} \left( \frac{M_t}{2.8 M_\odot} \right)^{-1} \left( \frac{a}{15500\text{km}} \right)^{4/3} \text{yr} \]

\[ N_{\text{cycle}} = 1.66 \times 10^7 \left( \frac{M_1}{1.4 M_\odot} \right)^{-1} \left( \frac{M_2}{1.4 M_\odot} \right)^{-1} \left( \frac{M_t}{2.8 M_\odot} \right)^{-1/2} \left( \frac{a}{15500\text{km}} \right)^{5/2} \]

\[ h_c = 1.45 \times 10^{-23} (1 + z)^{5/6} \left( \frac{M_c}{1.2 M_\odot} \right)^{5/6} \left( \frac{f}{0.1 \text{Hz}} \right)^{-1/6} \left( \frac{d_L}{10\text{Gpc}} \right)^{-1} \]

\[ \frac{d t_o}{d t_e} = \frac{a_o}{a_e} = (1 + z) \]

\[ \frac{d^2 t_o}{d t_e^2} = (1 + z) a_e^{-1} \left( \partial_t a(t_o) - \partial_t a(t_e) \right) \]

\[ \equiv g_{\cos}(z) = (1 + z)((1 + z)H_0 - H(z)) \]
\[ \Delta t_o = \Delta t_e (1 + z) + \frac{g_{\cos}(z)}{2} \Delta t_e^2 + \ldots \]

\[ \Delta t_o = \Delta T + X(z) \Delta T^2 + \ldots \]

with \( X(z) \equiv \frac{g_{\cos}(z)}{2(1 + z)^2} \)

\[ \Delta T = (1 + z) \Delta t_e \]

\( X(z) \) is positive, then \( \partial_t a(t_o) > \partial_t a(t_e) \)

Now for \( \Delta T \sim 10^9 \) sec

\[ \sim 10^{18}/(3 \times 10^{17}) \sim 1 \text{ [sec]} \]

\[ \Omega_{k0} = \left\{ 1 - \left( \frac{dr(z_s)}{dz} \right)^2 (1 + z_s)^2 (H_0 - 2X(z_s))^2 \right\} \left\{ r(z_s)^2 H_0^2 \right\}^{-1} \]

\[ r(z) \equiv \frac{d_L(z)}{(1 + z)} \]
Equation of state of the universe

- Even if $d_L(z)$ is known accurately as a function of $z$, for example by SNAP, the density and the pressure are not uniquely determined. (Weinberg 1970; Chiba and TN 1999)

- The value of $\Omega_{k0} \equiv \frac{k}{a_0^2 H_0^2}$ should be determined by other observations.

\[
\kappa^2 \rho(z) = 3 \left[ \frac{1}{(dr/dz)^2} + \left( (1+z)^2 - \frac{r^2}{(dr/dz)^2} \right) H_0^2 \Omega_{k0} \right], \quad r(z) = \frac{d_L(z)}{(1+z)}
\]

\[
\kappa^2 p(z) = -\frac{3}{(dr/dz)^2} + (1+z) \frac{d}{dz} \left( \frac{1}{(dr/dz)^2} \right) - \left[ (1+z)^2 - \frac{3r^2}{(dr/dz)^2} + (1+z) \frac{d}{dz} \left( \frac{r^2}{(dr/dz)^2} \right) \right]
\]
Matched Filter Analysis Using Ultimate DECIGO

sources at $z = 1$

- BH $10M_\odot$
- NS $1.4M_\odot$

$S_\text{NS}$

$10^6$

$10^5$

$10^4$

$\Delta t [\text{yr}]$

$\Delta \chi/t_0$

- 1
- 0.1
- 0.01
- 0.001

$\Delta t [\text{yr}]$
\[ \tilde{h}(f) = \int_{-\infty}^{\infty} e^{2\pi ift} h(t) dt \]

the stationary phase approximation

\[ \tilde{h}(f) = K d_L(z)^{-1} M_c^{5/6} f^{-7/6} \exp[i\Phi(f)] \]

K depends on directions and binary orientation

\[ \Phi(f) = 2\pi ft_c - \phi_c - \frac{\pi}{4} + \frac{3}{4} (8\pi M_{cz} f)^{-5/3} \]

\[-\frac{25}{32768} X(z) f^{-13/3} M_{cz}^{-10/3} \pi^{-13/3} \]

\[ M_{cz} = M_c (1 + z) \quad \text{Newtonian quadrupole formula} \]

six parameters

\[ \{ A, M_{cz}, \mu_z, t_c, \phi_c, M_{cz}^{-10/3} X(z) \} \]

\[ K d_L(z)^{-1} M_c^{5/6} \quad (1 + z)M_1 M_2 / M_t \]
\[ \Delta T = 16\text{yr} \]

\[ \Delta M_{cz}/M_{cz} = 1.5 \times 10^{-11} \]

\[ \Delta \mu_z/\mu_z = 4.2 \times 10^{-8} \]

\[ \Delta A/A \sim (S/N)^{-1} = 5 \times 10^{-5} \]

\[ z = d_L^{-1} \text{(distance)} \]

\[ M_1 \text{ and } M_2 \text{ for } \sim 10^5 \text{ binaries per year} \]

up to \( z = 1 \)

- After subtracting these binaries and possible other sources from the row data, we might observe
- the primordial gw background even if \( \Omega_{GW} \sim 10^{-20} \)
Punch Point of Ultimate DECIGO

a) 100,000 Mass of neutron stars, Black Hole will give us mass function of NS and BH

b) Direct measurement of Acceleration of the universe; Independent measurement of the curvature of the universe, independent information of EOS of the universe

c) Background gw predicted by inflation model up to $\Omega_{GW} \sim 10^{-20}$ Completely independent information from MAP and PLANCK

d) If the fundamental scale is Tev, then the redshifted GW at T=Tev is just 0.1Hz Band. We may see something.
Other Sources are also important

a) Formation of Intermediate mass BH (IMBH)
\[ M \sim 10^4 M_\odot \]

b) Coalescence of IMBH binary
c) Compact star falling into IMBH
d) GRB Jets
e) White dwarf binary
f) Oscillation of WD
g) WD+NS(BH) binary
h) .............
Various version of DECIGO is possible

A) the same spec as LISA with 0.01 arm length 1w laser, low acceleration noise

B) The ultimate one
   quantum limit;100kg mirror, 10MW laser

C) The practical one
   300w laser; 3m mirror…… similar to BBO (Big Bang Observer)
If we use the same spec as LISA

arm length $=$ 0.01 LISA
Spec of Practical DECIGO

- 300w laser, 3m mirror, 0.01 LISA acceleration noise........similar to BBO(Big Bang Observer)
- POINT is
- Practical DECIGO and ground based interferometer and bars can observe the same source at different frequency and time.
- Here we like to point out
- Deci Hertz laser interferometer can determine the position of the coalescing binary neutron stars within an arc minute a week before the final event to black hole (Takahashi and TN (2003) ApJ 596 L231)

- New binary pulsar suggests the nearest distance is ~ 50Mpc comparable to the nearest GRB980425(40Mpc) Accuracy can be ~ 10arcsec
Practical DECIGO or BBO

\[ \sqrt{\text{S}_n(f)} \text{ (Hz}^{-1/2}) \]

- LISA
- LIGO II
- LCGT
- NS+NS at 300 Mpc
- DECIGO/BBO

\( f \text{ (Hz)} \)
Angular Resolution

• In any astronomy including Gamma Ray Astronomy and Gravitational Wave Astronomy

Angular resolution is crucial

Good example is Gamma Ray Bursts 1973-1997 Distance was not determined.
2704 BATSE Gamma-Ray Bursts

angular resolution is > 1°
1997 Beppo-SAX satellite

- Afterglow of GRB in X-ray
- X-ray telescope with arc minute accuracy
- X-ray counter Part
- Optical telescope
- OT (Optical Transient)
- Optical Afterglow
- HOST Galaxy
- Spectrum
- Cosmological redshift
Cosmological GRB !!

\[ \frac{\lambda_{\text{obs}}}{\lambda_0} = 1 + z \]

- \( Z > 0.835 \) for GRB970508

- \( z = 0.768 \) \( 0.835 \)
Optical Afterglow and Host Galaxy

- GRB 970228
- GRB 990123

- GRB 970228

- GRB 990123
Distance to short GRBs is not known

- short GRB might be coalescing binary neutron star

Figure 1. The number of GRBs versus the duration ($T_{90}$), where the duration includes 90 per cent of the total GRB counts above 25 keV in the BATSE detectors. The two curves are lognormal fits to the data.
Angular Resolution in Gravitational Wave Astronomy

- Essentially time of flight method
  \[ \cos \theta = \frac{ct}{D} \quad \delta \theta \sim \frac{c\delta t}{D} \]

- For periodic or chirp signal
  \[ \delta \theta \sim \frac{\lambda}{D} \delta \Phi \quad \delta \Phi \text{ accuracy of phase} \]

- LIGO and LISA
  \[ D \sim \lambda \]
  so that \[ \delta \theta \sim \delta \Phi \sim S/N^{-1} \sim 0.01 \sim 1\text{degree} \] for \( S/N \sim 100 \)
How to determine the direction

- Time of Flight Method

\[
\cos \theta = \frac{ct}{D}
\]
<table>
<thead>
<tr>
<th>$T_{\text{obs}}$ (yr)</th>
<th>$\Delta A/A$</th>
<th>$T_{\text{obs}}\Delta f$</th>
<th>$T_{\text{obs}}^2\Delta f$</th>
<th>$\Delta \Omega_S$ (sr)</th>
<th>$\Delta \Omega_L$ (sr)</th>
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<tr>
<td>$f = 10^{-4}$ Hz</td>
<td></td>
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<tr>
<td>1 yr</td>
<td>0.205</td>
<td>0.33</td>
<td>0.59</td>
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<td>0.31</td>
<td>0.58</td>
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<td>0.154</td>
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<tr>
<td>10 yr</td>
<td>0.204</td>
<td>0.22</td>
<td>0.43</td>
<td>$6.78 \times 10^{-2}$</td>
<td>0.185</td>
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<td>$f = 10^{-3}$ Hz</td>
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<td>0.205</td>
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<tr>
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<td>0.204</td>
<td>0.22</td>
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<td>$2.70 \times 10^{-2}$</td>
<td>0.161</td>
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<td>1 yr</td>
<td>0.205</td>
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<td>10 yr</td>
<td>0.204</td>
<td>0.22</td>
<td>0.43</td>
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<td>0.110</td>
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<td>0.204</td>
<td>0.22</td>
<td>0.43</td>
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<td>0.109</td>
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</table>

Table 1: LiSA's measurement accuracy for binaries with angular parameters ($\cos \bar{\theta}_S = 0.3, \bar{\phi}_S = 5.0, \cos \bar{\theta}_L = -0.2, \bar{\phi}_L = 4.0$). Results are normalized by $SNR = 10$. Errors scale as $(SNR/10)^{-1}$ for $\Delta A, \Delta f$ and $\Delta f$, and $(SNR/10)^{-2}$ for $\Delta \Omega_{S,L}$. The second lines in each observational period $T_{\text{obs}}$ represent the case with removing the chirp signal $\dot{f}$ from fitting parameters and the third lines with removing the direction of the source ($\theta_0, \phi_0$).

for $S/N$ 100

$\delta\theta_S, \delta\phi_S \sim 1^\circ$

Practical DECIGO

\[ \lambda \sim 0.01D \]

\[ \delta \theta \sim 0.01S/N^{-1} \sim 1\text{arcmin} \]

is expected for S/N=100

- Consider 1.4 solar mass binary neutron star at 300Mpc

\[ f_{init} = 0.23 (M_x/M_\odot)^{-5/8} (T_{obs}/1\text{yr})^{-3/8}\text{Hz} \]
\[ \tilde{h}(f) = A f^{-7/6} e^{i\Psi(f)} \]

\[ A = K \sqrt{5/96} \mathcal{M}_z^{5/6} / (\pi^{2/3} D_S) \]

eight parameters \( (\mathcal{M}_z, \mu_z, \beta, \phi_c, t_c, D_S, \theta_S, \phi_S) \)

\[ \langle \Delta \gamma_i \Delta \gamma_j \rangle = (\Gamma^{-1})_{ij} \]

\[ \Gamma_{ij} = 4 \text{Re} \int \frac{df}{Sn(f)} \frac{\partial \tilde{h}^*_D(f)}{\partial \gamma_i} \frac{\partial \tilde{h}_D(f)}{\partial \gamma_j} \]

\[ \tilde{h}_D(f) = \tilde{h}(f) e^{i\phi_D(f)} \]

\( \phi_D(f) \) is the Doppler phase

\[ \phi_D = 2\pi f R \sin \theta_S \cos(2\pi t / T - \phi_S) \]

\[ R = 1 \text{ AU} \quad T = 1 \text{ yr} \]

\( (\theta_S, \phi_S) \) the direction to the source
\[(S/N)^2 = 4 \int \frac{df}{S_n(f)} \left| \tilde{h}_D(f) \right|^2\]

- We consider NS binary (1.4 \( M_\odot \)) and BH binary (10-1000) \( M_\odot \) at 300Mpc for 1yr and 10yr observation.
- The errors scale as \((S/N)^{-1}\) for the change of the distance and the orientation of the binary.
- The accuracy of the chirp mass and the reduced mass are \(10^{-7}\) and \(10^{-4}\) for 1.4 \( M_\odot \) NS binary, respectively.

The distance to the binary is determined by \((S/N)^{-1}\).
<table>
<thead>
<tr>
<th>Binary Masses ($M_\odot$)</th>
<th>S/N</th>
<th>$\Delta M_z / M_z$</th>
<th>$\Delta \mu_z / \mu_z$</th>
<th>$\Delta t_c$ (sec)</th>
<th>$\Delta D_S / D_S$</th>
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<td>1.4 + 1.4</td>
<td>62</td>
<td>$2.0 \times 10^{-7}$</td>
<td>$7.4 \times 10^{-4}$</td>
<td>$6.6 \times 10^{-2}$</td>
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<td>$8.8 \times 10^{-3}$</td>
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<tr>
<td>10 + 10</td>
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<td>$10^2 + 10^2$</td>
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<td>0.47</td>
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</table>

**Table 1**

The estimation errors of the chirp mass $M_z$, the reduced mass $\mu_z$, the coalescence time $t_c$ and the to the source $D_S$. The results are presented for the various binary masses ($1.4, 10^{1-3}M_\odot$) for 1yr (the upper line) and 10 yr (the lower line) observation.
Fig. 2.— The relative probability distribution of the angular resolution of DECCIGO/BBO in the case of 1 yr observation. For each mass case we randomly distribute $10^4$ binaries on the celestial sphere at $D_S$ (the Hubble parameter $h = 0.7$), and we show the probability distribution of the angular resolution for these sources. The solid lines and the dashed lines show $\Delta \theta_S$ and $\Delta \phi_S$, respectively. The signal to noise ratio $S/N$ and the estimation error in the coalescence time $\Delta t_c$ are also shown. $S/N$ is independent of $\theta_S$ and $\phi_S$ since the phase
This can be 5 times better sensitivity (∼ 10 arcsec) with new coalescence rate
Point All the Detectors to coalescing binary neutron star (black hole) event!!

- The direction as well as the time of the event are known beforehand
- All band electromagnetic detectors from radio to ultrahigh energy gamma rays
- Possible neutrino detectors
- Tune the high frequency gravitational wave detectors to catch ISCO, QNM and so on
• Recent discovery of new binary pulsar PSR J0737-3039
• Coalescence rate 180/Myr/Galaxy
• The nearest event in a year is 50Mpc
• S/N increases by a factor 6
• Then the position accuracy becomes 10arcsec and 0.01sec

Conclusion

• DECIGO/BBO can obtain
  • 100,000 mass of neutron stars and black holes
  • Direct measurement of the acceleration of the universe
  • Background gravitational waves
  • Other sources such as IMBH…….
• The angular position and the time of the coalescence a week before with 10arcsec and 0.01 sec accuracy
• Point all possible detectors to the source and Possible identification of short duration GRB