$f(R)$ gravities and PPN constraints

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In 12 **minutes . . .**

- General properties of $f(R)$ gravities in **Metric** and **Palatini** formalisms.

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}f(R) + S_m[g_{\mu\nu}, \psi_m]$$

$$\left[f(R) = R - \frac{\mu^4}{R};\ f(R) = R + \lambda R^2;\ f(R) = R + a\log R; \ldots\right]$$

Metric form → $(g_{\mu\nu})$ **Palatini form** → $(g_{\mu\nu}, \Gamma^{\alpha}_{\mu\nu})$

- Some constraints on these theories from solar system experiments.

Manipulations of the action

The action

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + S_m[g_{\mu\nu}, \psi_m]$$

is equivalent to

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \Big[f(\Phi) + f'(\Phi)(R - \Phi) \Big] + S_m[g, \psi]$$

where the EOM for Φ is $\Phi = R$.

$\Rightarrow$ $f(R)$ gravities are **Scalar-Tensor** theories.

Scalar-Tensor Representation

$f(R)$ have a S-T representation of the form

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[\phi R - \frac{w(\phi)}{\phi}(\partial_\mu\phi\partial^\mu\phi) - V(\phi)\right] + S_m[g_{\mu\nu}, \psi_m]$$

where

- $\phi \equiv \frac{8\pi}{\kappa^2} f'$ with $f' \equiv \frac{df(\Phi)}{d\Phi}$
- $V(\phi) = \frac{8\pi}{\kappa^2}(\Phi f'(\Phi) - f(\Phi))$

and

$$\begin{array}{lcl} \textbf{Metric form} & \Rightarrow & \omega = 0 \\ \textbf{Palatini form} & \Rightarrow & \omega = -\frac{3}{2} \end{array}$$

E.O.M. of scalar-tensor theories

For the metric:

$$\begin{aligned} G_{\alpha\beta} &= \frac{8\pi}{\phi}T_{\alpha\beta} - \frac{V}{2\phi}g_{\alpha\beta} + \frac{\omega}{\phi^2}\left(\partial_\alpha\phi\partial_\beta\phi - \frac{1}{2}g_{\alpha\beta}(\partial\phi)^2\right) + \\ &+ \frac{1}{\phi}\left(\nabla_\alpha\nabla_\beta\phi - g_{\alpha\beta}\Box\phi\right) \end{aligned}$$

For the scalar field:

$$[3+2\omega(\phi)]\Box\phi + 2V(\phi) - \phi V'(\phi) = 8\pi T - \overbrace{\omega'(\phi)\left(\partial_\mu\phi\partial^\mu\phi\right)}$$

Metric form of $f(R)$. I

- If V can be expanded around a minimum, $\varphi \equiv \phi - \phi_0$ satisfies

$$(\Box - \mu^2)\varphi = -\frac{8\pi\rho}{3+2\omega} - \frac{2V_0}{3+2\omega}$$

Post-Newtonian limits:

$$\begin{aligned}
\varphi &= \frac{2}{3+2\omega}\left(\frac{M}{r}e^{-\mu r} + \frac{V_0}{\mu^2}\right) \\
g_{00} &= -1 + \frac{2}{a^2\phi_0}\frac{M}{r}\left[1 + \frac{e^{-\mu r}}{3+2\omega}\right] + \frac{V_0}{6\phi_0}r^2 \\
g_{ij} &= \left(1 + \frac{2}{a^2\phi_0}\frac{M}{r}\left[1 - \frac{e^{-\mu r}}{3+2\omega}\right] - \frac{V_0}{6\phi_0}r^2\right)\delta_{ij}
\end{aligned}$$

- $\mu r \ll 1$ → Brans-Dicke constraints ($\omega > 10^4$).

- $\mu r \gg 1$ → General Relativity $\forall\ \omega$.

Metric form of $f(R)$. III

- $\mu r \gg 1$ for $\omega = 0$ is disturbing, since

$$R = V' + \frac{\omega}{\phi^2}(\partial\phi)^2 - \frac{2\omega}{\phi}\Box\phi$$

  at $\omega = 0$ becomes $R = V' \approx V_0''\varphi$

$$R \to \frac{2V_0}{\phi_0} \text{ everywhere}$$

- In these models the matter does not generate spacetime curvature!!!.

- A model with $V' \approx 8\pi\rho$ would imply an important self-interaction $V(\phi)$ that could strongly affect the vaccuum solutions of the metric.

Palatini form of $f(R)$. I

- The E.O.M. for ϕ is

$$2V(\phi) - \phi V'(\phi) = 8\pi T$$

- Outside the matter sources, $T = 0$,

$$\phi = \phi_0 = \text{ constant}$$

$$G_{\alpha\beta} = \frac{8\pi}{\phi_0} T^{rad}_{\alpha\beta} - \frac{V_0}{2\phi_0} g_{\alpha\beta}$$

- For spherical, charged bodies

$$\begin{aligned} ds^2 &= -A(r)dt^2 + \frac{1}{A(r)}dr^2 + r^2 d\Omega^2 \\ A(r) &= 1 - \frac{2C}{r} + \frac{Q^2}{\phi_0 r^2} - \frac{V_0}{6\phi_0} r^2 \end{aligned}$$

Palatini form of $f(R)$. II

According to

$$A(r) = 1 - \frac{2C}{r} + \frac{Q^2}{\phi_0 r^2} - \frac{\Lambda r^2}{3}$$

- For neutral bodies, the Newtonian limit, $C = GM/c^2$, is recovered if the $\Lambda = \frac{V_0}{2\phi_0}$ term is negligible in solar system scales.

- For charged bodies, assuming Λ negligible, $C = GM/c^2$ is valid if ϕ_0^{-1} is not abnormally large.

→ The only constraint from solar system experiments is the value of Λ.

Example: $f(R) = R - \frac{\mu^4}{R}$

In this theory, the potential is $V = \frac{2\mu^2 c^3}{G}\sqrt{\tilde{\phi} - 1}$ and

$$2V - \phi V' = \frac{\mu^2 c^3}{G}\left(\frac{4 - 3\tilde{\phi}}{\sqrt{\tilde{\phi} - 1}}\right)$$

where $\tilde{\phi} = \frac{G}{c^3}\phi$.

Metric form: $3\Box\phi + \overbrace{2V - \phi V'} = 8\pi T$

→ The self-interaction is negligible
→ Brans-Dicke theory → Non-viable.

Palatini form: Defining $\rho_\mu \equiv \frac{\mu^2 c^2}{8\pi G} \sim 10^{-26} g/cm^3$, then the algebraic equation $2V - \phi V' = -8\pi\rho$ leads to

$$\frac{G}{c^3}\phi = \begin{cases} 1 + \left(\frac{\rho_\mu}{\rho}\right)^2 + O(4) & \text{inside matter} \\ \frac{4}{3} & \text{outside matter} \end{cases}$$

→ Equivalent to GR inside massive bodies
→ Tiny cosmological constant in vaccuum

Conclusions

- **Metric form of $f(R)$:**
  - Non-viable when $2V - \phi V' \ll 8\pi\rho$
  - "Disturbing" interpretation of R
- **Palatini form of $f(R)$:**
  - Same vaccuum behavior as GR up to a cosmological constant term.
  - Only inside massive media the dynamics is modified with respect to GR.
  - They represent reasonably good generalizations of GR in comparison with other scalar-tensor theories.