To the Allegro and Explorer Collaboration

Dear Professors Coccia, Hamilton, Johnson, and Pizzella,

We are writing to you in connection with the paper published in the 19 May 1999 issue of Physical Review D\(^1\), giving upper limits on gravitational wave burst sources, based on a coincidence analysis of six months of Allegro and Explorer data from 1991. If you wish, please feel free to make this letter available to your collaborators in this work, and the co-authors of your paper.

We are interested in finding out what limit would be placed on the rate of binary neutron star inspirals in the Galaxy by analysis of your data. To do this requires a model of Galactic inspiral sources. We have constructed a model of this kind for our recently posted paper setting upper limits using 1994 data from the LIGO 40-m prototype. Our paper may be found at http://xxx.lanl.gov/abs/gr-qc/9903108.

Of course what is really of the most interest here is to understand how to set limits like this in a rigorous way. We have been working on this issue for (coincident) interferometric data, and are interested in seeing how it might work for bar data as well.

In order to stimulate some discussion of these issues and perhaps to initiate a small project together, we have written some code to simulate signals from Galactic inspiral in the Allegro and Explorer detectors. This is based on exactly the same model of the Galactic distribution of binaries used in our paper cited above. This paper contains a mathematical and physical description of the assumed distribution, so we won’t spend any time describing it here.

Our code produces as output a list of inspiral events. In constructing the table, we have assumed that the distribution (in time) of the binaries is a Poisson process, with a mean rate of one inspiral every 100 seconds. Of course you can sub-sample our table to simulate inspiral rates lower than this. For example if you choose 20% of the members of the table at random, this would correspond to a simulated rate of one inspiral per 500 seconds, and so on\(^2\).

The table is a text file, with one line per simulated inspiral event. The table contains


\(^2\)I’ve had some recent discussions with M.A. Papa about this. For convenience in some analysis, we thought you might also find it helpful to have a different realization where the simulated events occur at regular intervals, once every hundred seconds, so we have provided a separate table of this type as well.
\[ N = 155,115 \] simulated events, covering UTC days 170 to 350 in 1991, the same period of time analyzed in your paper. Each line lists the following:

- The UTC time of the signal. Since binary inspiral chirps pass through the sensitive band of your detectors very quickly compared to your timing resolution or to the light-travel-time between the Allegro and Explorer detectors, only a single time is given. For your convenience we list this time in three formats:
  1. The UTC date (year=1991, day=1 to 365, HH:MM:SS).
  2. The number of seconds after 1 Jan 1970 (Unix time definition).
  3. The Modified Julian Day (MJD).

All three timestamps are equivalent: we were not sure which would be the most useful so provided all of them. Note that as the Earth rotates and the bars change their orientation relative to the Galactic center, the properties of the simulated signals change with time.

- The dimensionless quantities \( \frac{|\tilde{H}(f)|}{\tau} \) and \( \arg \tilde{H}(f) \) (range 0 to 2\( \pi \)) for each of the two bar modes at each of the two sites. As in your paper, \( \tau = 10^{-3} \) sec.
  We list these in the following order:
  1. \( |\tilde{H}(f)|/\tau \) and \( \arg \tilde{H}(f) \) for the Allegro \( f = 896.7 \) Hz mode.
  2. \( |\tilde{H}(f)|/\tau \) and \( \arg \tilde{H}(f) \) for the Allegro \( f = 920.2 \) Hz mode.
  3. \( |\tilde{H}(f)|/\tau \) and \( \arg \tilde{H}(f) \) for the Explorer \( f = 904.7 \) Hz mode.
  4. \( |\tilde{H}(f)|/\tau \) and \( \arg \tilde{H}(f) \) for the Explorer \( f = 921.3 \) Hz mode.

We have used the site location and bar orientation information from the 19 May 1999 paper to determine the strain that would be produced in each bar\(^3\). Although the bar axes are only misaligned by about 12 degrees, there is in some cases a large difference in these amplitudes between the two bars (though very little difference in amplitude between the two modes of a given bar)\(^4\). Our Fourier-transform convention\(^5\) is that

\[
\tilde{H}(f) = |\tilde{H}(f)| \exp(i \arg \tilde{H}) = \int_{-\infty}^{\infty} h(t) \exp(2\pi i ft) \, dt
\]

and we specify frequency \( f \) in Hz. We were not sure if you had any need for (or use for) the phase of \( \tilde{H}(f) \) but provided it just in case it might be needed.

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\(^3\)An important concern what is meant by \( h(t) \). We define \( h(t) = \frac{1}{2} \frac{\Delta L(t)}{L(t)} \), where \( L \) is the average total length of the bar and \( L + \Delta L(t) \) is the actual total length of the bar at time \( t \). This agrees with the conventions of Thorne’s article in 300 Years of Gravitation, Eqn. (6). If these are not your conventions, please tell us - we are worried that in the bar community the factor of 1/2 may be left out in defining \( h(t) \).

\(^4\)This happens if either one bar axis happens to align with the direction to the source, or if the projection of the orbital plane onto the sky is a line (degenerate ellipse) and the projection of one bar’s axis onto the sky is orthogonal to it.

\(^5\)We were not sure if there were factors of \( \sqrt{2\pi} \) in the definition of the \( H(\omega) \) that appears in Eqn. (1) of your 19 May paper. M.A. Papa informed me that this is the definition that you used, but it should be checked. If not, then there are missing powers of \( \sqrt{2\pi} \) - please let us know in this case!
Similarly, we doubt that the difference in $|\tilde{H}(f)|/\tau$ and $\text{arg}\,\tilde{H}(f)$ at the two different mode frequencies of a given bar matters, but again we give both just in case it might be needed.

We found the simple method that you used to analyze the data set quite persuasive. The idea of shifting the two data sets by all possible time lags and using this to determine (experimentally) the probability distribution of false coincidences is a nice one. We wanted to suggest that you might start using the simulated signal list that we have provided as follows:

- Begin with the original data sets used for your 19 May, 1999 paper.
- Examine the inspirals which we have provided in our list, and ask, for each inspiral, which ones would have registed as events in the outputs of both of your detectors, i.e. which ones would have appeared in both Fig. 1 and Fig. 2 of your 19 May paper. As we understand it, all the events of the Explorer list with burst amplitude of greater than $2.3 \times 10^{-18}$ should be included during all operational times. For the Allegro detector, all events exceeding a burst amplitude that varies with the detector performance (but is typically $3 \times 10^{-18}$) should be included.
- Let’s call the fraction of events on our list that would have appeared as events in both detectors $f$. These are events that exceed the needed thresholds, and occur during times that both Allegro and Explorer are in operation.
- We assume that if the number of inserted simulated inspiral events is small (say, less than a few hundred), it will not have much effect on the probability of accidentals distribution curves of Fig. 5. Once you have settled on an upper limit for the rate $R$ (below) someone should go back and verify that this statement is true.
- Now let’s ask for the number of coincidences needed to exceed by some reasonable number of standard deviations (say 4σ) the mean number of accidentals in Fig. 5. This should be about (reading from the Figure) $17 + 4 \times 8 = 52$ for the 0.29 s coincidence window or $60 + 4 \times 10 = 100$ for the 1.00 s coincidence window. Let’s denote the number of coincidences needed to establish detection with some stated confidence $N_{4\sigma}$. As we said, it appears to us that this number is about 50 to 100.
- From the fraction $f$ of inspiral events on our list that pass the detector thresholds and the number $N_{4\sigma}$ needed to get some number of standard deviations to the right of the mean number of accidental coincidences, it should be possible to infer an upper bound on the Galactic inspiral rate $R$ with some stated confidence. This would be

$$R_{4\sigma} = (100 \text{ sec})^{-1} \left( \frac{N_{4\sigma}}{fN} \right)$$

where $N$ is the total number of events on our list.

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6 We have not thought carefully about how to relate this threshold to the false alarm and false dismissal rates, and imagine that you have already thought carefully about this, so we’ll be sloppy for the moment.

7 We would give these numbers more precisely but the necessary information is only in your 19 May paper in Fig. 5, so we are just doing a rough estimate from this published graph.
It is not clear to us that this is the “best” way to analyze your data set, but it preserves the simplicity of the methods which you have already used, and should be a simple way to start. We think that this would be interesting to do, and a useful way for us to understand the issues involved in coincidence analysis a bit better. Bruce Allen will be in Rome from June 14-18, so we can discuss this further at that time.

We have posted some materials for you at our web site.

- List of simulated events (mean rate \(10^{-2} \text{/sec, Poisson distributed}\):  
  www.lsc-group.phys.uwm.edu/~jolien/barevents.out.gz

- List of simulated events at equal 100 second intervals:  
  www.lsc-group.phys.uwm.edu/~jolien/barevents-equal.out.gz

- Code that produced them:  
  www.lsc-group.phys.uwm.edu/~jolien/barevents.c

- This letter:  
  www.lsc-group.phys.uwm.edu/~jolien/barevents.ps

We hope that you find these interesting and perhaps useful.

Sincerely,

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