To find a burst signal in burst-filled noise

OVERVIEW

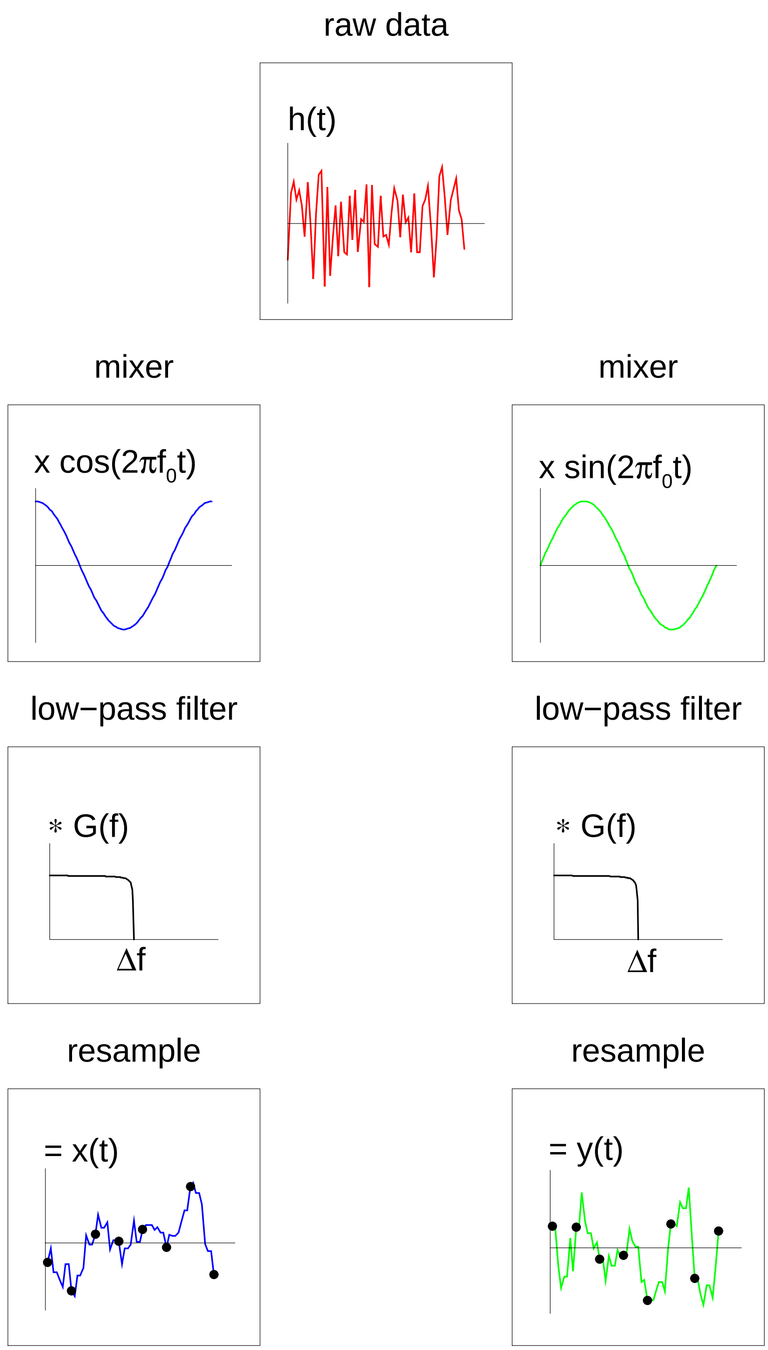
I Signal model

- Signal is a burst with unknown form but with known duration and frequency band
- Duration of signal is Δt
- Frequency bandwidth is Δf about a central frequency f_0
- Signal has unity time-frequency volume

$$V = \Delta t \times \Delta f = 1$$

  (e.g., a ringdown)
- Signal has unknown amplitude A (single-detector matched filter SNR)

II Data conditioning



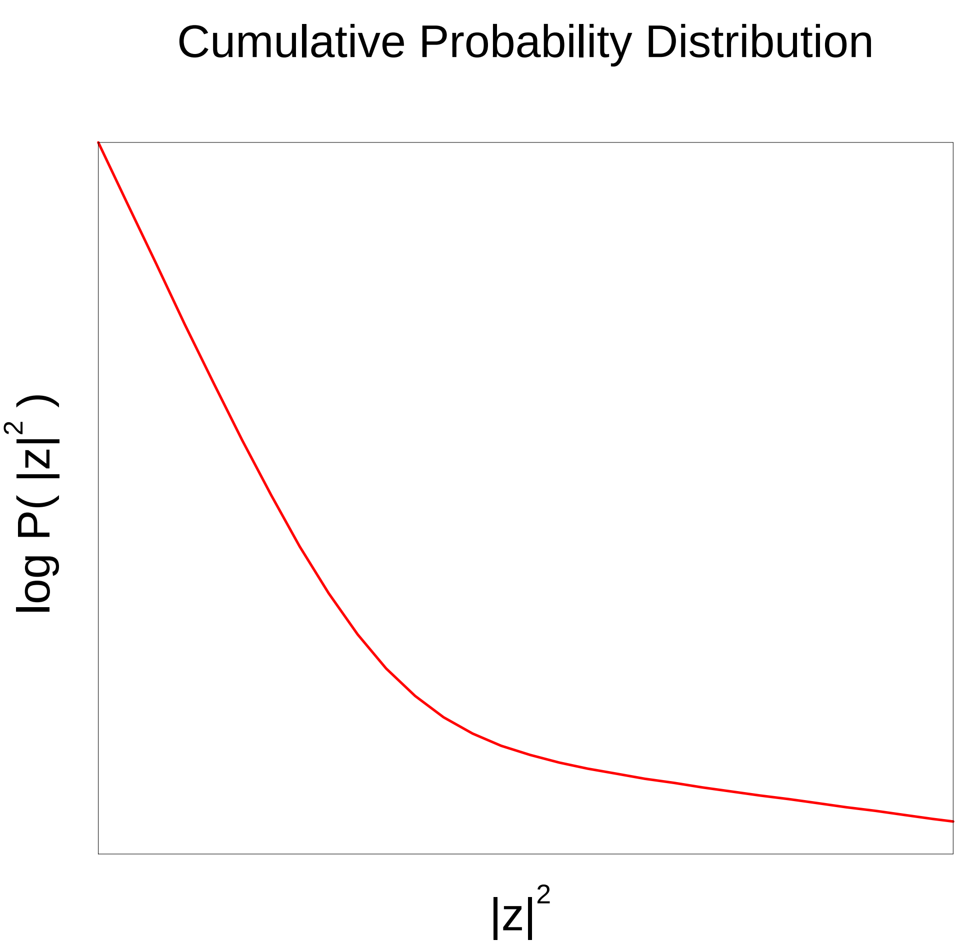
Output is $z(t) = x(t) + iy(t)$
Power statistic is $|z(t)|^2 = x^2(t) + y^2(t)$
(optimal for Gaussian noise)

III Noise model

Noise has probability distribution

$$p(x,y) = \frac{1-P}{2\pi} e^{-x^2/2} e^{-y^2/2} + \frac{P}{2\pi\sigma^2} e^{-x^2/2\sigma^2} e^{-y^2/2\sigma^2}$$

where P is the probability of the burst
and σ is the burst amplitude scale



IV Detection statistics

- Assume there are two aligned detectors with the same sensitivity and identical but independent noise
- Total power statistic (optimal for Gaussian noise)

$$|z_1 + z_2|^2 = (x_1 + x_2)^2 + (y_1 + y_2)^2$$

- Coincidence power statistic

$$\min(|z_1|^2, |z_2|^2) = \min(x_1^2 + y_1^2, x_2^2 + y_2^2)$$

- Cross-correlation statistic

$$\mathrm{Re}(z_1^* z_2) = x_1 x_2 + y_1 y_2$$

- Locally optimal statistic

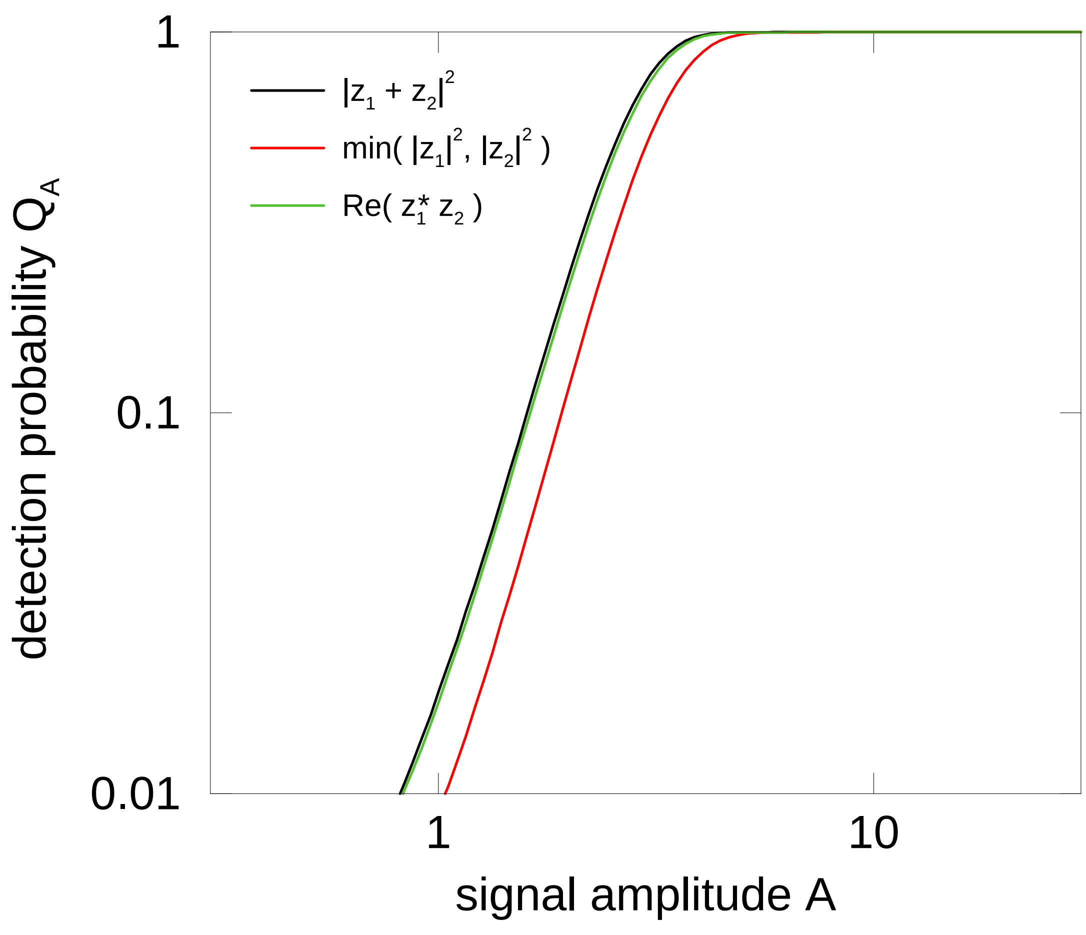
$$\simeq \begin{cases} |z_1 + z_2|^2 - 2 & \text{if } |z_1|^2, |z_2|^2 < Z^2 \\ |z_1|^2 - 1 & \text{if only } |z_1|^2 < Z^2 \\ |z_2|^2 - 1 & \text{if only } |z_2|^2 < Z^2 \\ 0 & \text{otherwise} \end{cases}$$

$Z^2 \simeq 2 \log(\sigma^2 / P)$

Va Performance results — Gaussian noise

Gaussian noise ($P = 0$)

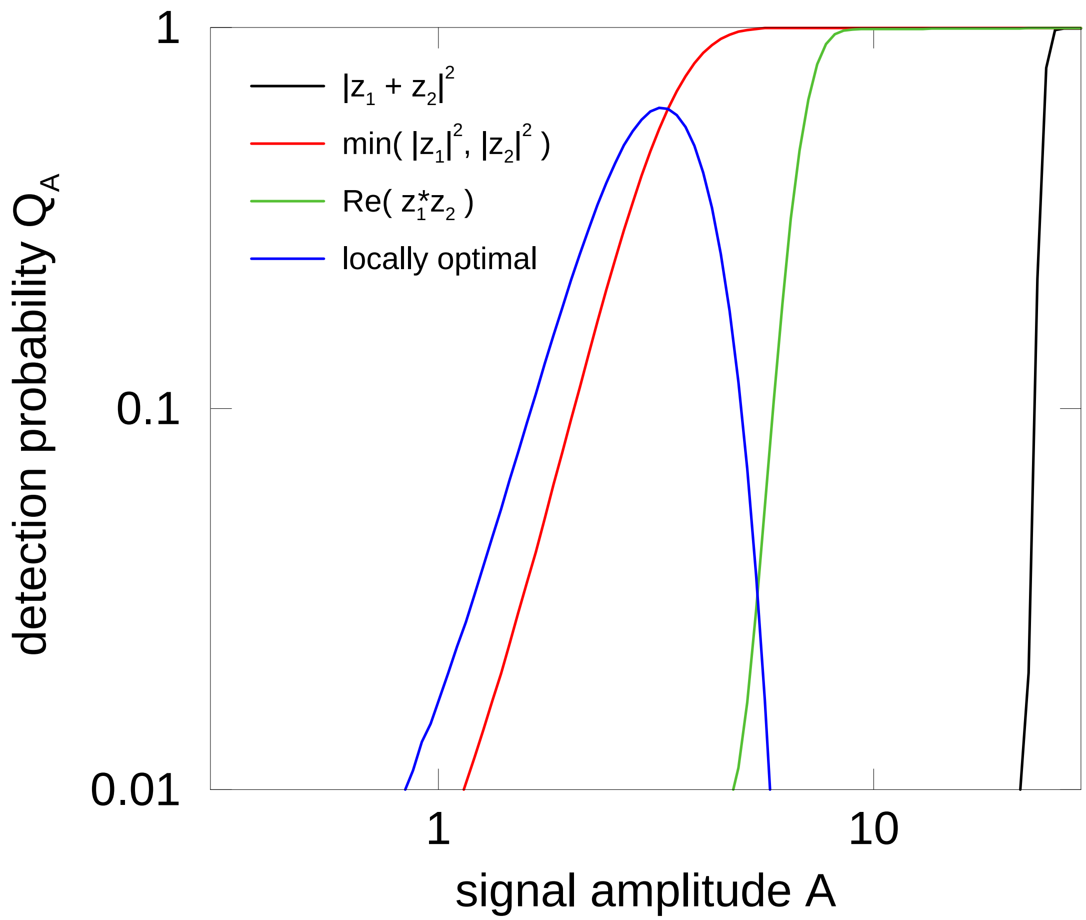
False alarm probability $Q_0 = 0.001$



Vb Performance results — non-Gaussian noise

Non-Gaussian noise ($P = 0.01$ and $\sigma = 20$)

False alarm probability $Q_0 = 0.001$



VI Conclusions

- ◇ Locally optimal statistic works well when noise is well-characterized but needs to be modified for detection of large amplitude signals
- ◇ Coincidence statistic is robust and is best when noise is not well characterized and $P^2 \ll Q_0 \ll P$
- ◇ Gaussian optimal statistic works best when $P \ll Q_0$

Future extensions

- ◇ signals that have larger time-frequency volume
- ◇ detectors that have different sensitivity
- ◇ more than two detectors
- ◇ detectors that are not aligned