Let us consider the calibrated noise at the 3 detectors $h_1(t)$, $h_2(t)$, $\ell_1(t)$

Assumption 1: the 3 noise time series are (a) uncorrelated, (b) stationary stochastic processes characterized by their mean and autocorrelation functions:

$$\langle h_1(t)\rangle = \langle h_2(t)\rangle = \langle \ell_1(t)\rangle = 0 \quad (1$$

$$\langle h_1(t)\, h_2(t)\rangle = \langle h_1(t)\rangle\langle h_2(t)\rangle = 0 \quad (2$$

$$\langle h_1(t)\, e_1(t)\rangle = \langle h_1(t)\rangle\langle e_1(t)\rangle = 0 \quad (3$$

$$\langle h_2(t)\, e_1(t)\rangle = \langle h_2(t)\rangle\langle e_1(t)\rangle = 0 \quad (4$$

$$\langle h_1(t)\, h_1(t+\tau)\rangle = R_1(\tau) \quad (5$$

$$\langle h_2(t)\, h_2(t+\tau)\rangle = R_2(\tau) \quad (6$$

$$\langle \ell_1(t)\, e_1(t+\tau)\rangle = R_3(\tau) \quad (7$$

the analysis here can be generalized to include a $\neq 0$ results in (2.

$$R_1(\tau) = \int_{-\infty}^{+\infty} e^{-i2\pi f t}\, S_{H1}(f)\, df \quad (8$$

$$R_2(\tau) = \int_{-\infty}^{+\infty} e^{-i2\pi f t}\, S_{H2}(f)\, df \quad (9$$

$$R_3(\tau) = \int e^{-i2\pi f t}\, S_{e_1}(f)\, df \quad (10$$

where $S_{H1}(f)$, $S_{H2}(f)$ $S_{e_1}(f)$ are the available noise power spectral densities.

CorrPower computes the r-statistics over 3 different coincidence windows T_1 T_2 T_3 and for each of them evaluates:

$$r_T = \frac{\int_{-\frac{T}{2}}^{\frac{T}{2}} x_\alpha(t)\, x_\beta(t+\tau)\, dt}{\| x_\alpha(t) \| \; \| x_\beta(t+\tau) \|} \qquad (11$$

where $\alpha, \beta = 1, 2, .., 3$ and $\qquad (12$

$$N_\alpha = \| x_\alpha(t) \| = \sqrt{\int_{-\frac{T}{2}}^{+\frac{T}{2}} x_\alpha(t)^2\, dt} \qquad (13$$

I assume that the energy in the noise does not depends on the exact location of the temporal window and that is simply proportional to the duration of the window.

$$N_\alpha = T \cdot n_\alpha \qquad N_\beta\ T\, n_\beta \qquad (14$$

I also assume that N_α and N_β are not random variables.

A first ingredient to evaluate the probability distribution for the r evaluated by corr power is to evaluate the pdf of

$$\bar{r}_T = \int_{-\frac{T}{2}}^{\frac{T}{2}} x_\alpha(t)\, x_\beta(t+\tau)\, dt = \qquad (15$$

$$= \sum_{i=1}^{N^2} x_\alpha(t_i)\, x_\beta(t_i+\tau)\, \Delta t \qquad (16$$

ASSUMPTION: N is large enough that the central limit theorem applies to (16.

$\bar{r}_T$ becomes Gaussian with mean & variances given by:

$$\langle \bar{r}_T \rangle = \sum_{i=1}^{N} \langle \cancel{X_\alpha(t_i)} \rangle^{0} \langle \cancel{X_\beta(t_i+\tau)} \rangle^{0} \Delta t = 0 \qquad (17$$

$$\sigma_{T_1}^2 = \langle \bar{r}_T^2 \rangle = \sum_{i=1}^{N} \sum_{j=1}^{N} \langle X_\alpha(t_i) X_\beta(t_i+\tau) X_\alpha(t_j) X_\beta(t_j+\tau) \rangle =$$

$$= \sum_{i=1}^{N} \sum_{j=1}^{N} \langle X_\alpha(t_i) X_\alpha(t_j) \rangle \langle X_\beta(t_i+\tau) X_\beta(t_j+\tau) \rangle$$

$$= \sum_{\substack{i=1\\ j=1}}^{N} R_\alpha(t_i - t_j)\, R_\beta(t_i - t_j) \qquad (18$$

the probability of r_{T1} becomes:

$$p(r_{T_1}) = \frac{1}{\sqrt{2\pi \sigma_{T_1}^2 N_\alpha^2 N_\beta^2}} \exp\left\{ \frac{r_{T1}^2}{2\sigma_T^2 N_\alpha^2 N_\beta^2} \right\}. \qquad (19$$

the probability for the CorrPower I

$$P(I) = P(I = \max\{ r_{T1}, r_{T2}, r_{T3} \}) = \qquad (20$$

$$P(r_{T_1}, r_{T_2}, r_{T_3}) = \frac{1}{\sqrt{2\pi \det C}} \exp\left\{ - \vec{r}\, C^{-1} \vec{r} \right\} \qquad (21$$

with $\vec{r} = (r_{T1}, r_{T2}, r_{T3})$ (22

and

$$C = \begin{pmatrix} \sigma_{T1}^2 & \sigma_{T1}^2 & \sigma_{T1}^2 \\ \sigma_{T_1}^2 & \sigma_{T_1}^2 + \Delta_1^2 & \sigma_{T1}^2 + \Delta_1^2 \\ \sigma_{T1}^2 & \sigma_{T1}^2 + \Delta_1^2 & \sigma_{T1}^2 + \Delta_2^2 \end{pmatrix} \qquad (23$$

$$P(\Gamma = r_M) = \frac{1}{3}\Big(P(r_{T_1} = \Gamma,\ r_{T_2} < \Gamma,\ r_{T_3} < \Gamma) + P(r_{T_1} < \Gamma,\ r_{T_2} = \Gamma,\ r_{T_3} < \Gamma) + P(r_{T_1} < \Gamma,\ r_{T_2} < \Gamma,\ r_{T_3} = \Gamma) \Big) \quad (24$$

In deriving (23 it is assumed that (a) the most correlated T_1 window is included in the most correlated T_2 window and (b) the most correlated T_2 window is included in the most correlated T_3 window and (c) that

$$\langle r_{T_1} r_{T_2} \rangle = \sigma_{T_1}^2 \quad (25$$

$$\langle r_{T_2} r_{T_3} \rangle = \sigma_{T_1}^2 + \Delta_1^2 \quad (26$$

$$\langle r_{T_1} r_{T_3} \rangle = \sigma_{T_1}^2 \quad (27$$

$$\langle r_{T_2} r_{T_2} \rangle = \sigma_{T_1}^2 + \Delta_1^2 \quad (28$$

$$\langle r_{T_3} r_{T_3} \rangle = \sigma_{T_1}^2 + \Delta_2^2 \quad (29$$

the assumptions above require a correlation time of few periods.

Eq 24 requires integrating (22 for 2 of the 3 random variables.

$$\Delta_1^2 = \langle r_{T_\alpha}^2 \rangle \quad \text{where} \quad T_\alpha = T_2 - T_1$$

$$\Delta_2^2 = \langle r_{T_\beta}^2 \rangle \quad \text{where} \quad T_\beta = T_3 - T_1$$

$p\,(T = r_M,\ \gamma_{NONOISE})$ follow the same path from

$$\vec{\gamma} = (\gamma, \gamma, \gamma)$$

$$P[\vec{r}, \gamma] = \frac{1}{\sqrt{2\pi \det C}} \exp\left\{ -(\vec{r} - \vec{\gamma})\, C^{-1} (\vec{r} - \vec{\gamma}) \right\}$$