Conventions for data and software products of the LIGO and the LSC

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1 Preamble

This technical note addresses the need for uniform conventions across the multiple software packages being developed by the LSC. The need for concrete conventions is clear if different components of the software are to function together effectively and correctly. The \LaTeX{} source for this document is in under CVS and may be obtained as follows:

```
export CVSROOT=":pserver:anonymous@gravity.phys.uwm.edu:/usr/local/cvs/lscdocs"
cvs login
cvs co T/T010095
```

The password is lscdocs.

1.1 Scope of the conventions [REQUIRED]

- The conventions in this document will apply to all data and software products within the LIGO Scientific Collaboration and the LIGO Laboratory.
- This is a living document. A note in each section heading will indicate whether the recommended convention has been adopted by the LSC and the LIGO Lab:
  
  **REQUIRED** Indicates that the conventions discussed in this section have been considered by the Software Coordinator, the LSC and the Lab Directorate and have been adopted as the standard convention.
  
  **PENDING** Indicates that a recommendation has been forwarded to the software coordinator for consideration.
  
  **RECOMMENDED** Indicates that these conventions are being adopted by some subset of the collaboration and would be usefully adopted by all.
- Changes to **REQUIRED** conventions must be forwarded to the Software Coordinator as a formal change request as described in the charter of the Software Change Control Board.

2 Conventions for Discrete Fourier Transforms [REQUIRED]

2.0 Continuous Fourier Transforms

In the definitions of discrete Fourier transforms (DFT) it will be useful to refer the continuous Fourier transform (CFT) of a time domain quantity \( h(t) \):

\[
\hat{h}(f) = \int_{-\infty}^{\infty} dt \ h(t) \ e^{-i2\pi f(t-t_0)} ;
\]

the inverse operation is given by

\[
h(t) = \int_{-\infty}^{\infty} df \ \hat{h}(f) \ e^{i2\pi f(t-t_0)} .
\]

Additionally, a real function of time can be *heterodyned* by multiplying it by a complex exponential (reference signal):

\[
h_h(t) = h(t) \ e^{-i[2\pi f_h(t-t_0)+\varphi_0]} ,
\]

where \( f_h \) is known as the heterodyning base frequency and \( \varphi_0 \) is the phase of the reference signal at the origin \( t_0 \) of time. The CFT of the complex time series \( h_h(t) \) is related to the CFT of the original real time series \( h(t) \) by

\[
\hat{h}_h(f) = \hat{h}(f_h + f) \ e^{-i\varphi_0} .
\]
2.1 Forward and reverse DFT

Consider a time domain quantity $h(t)$ sampled at $N$ discrete points with sampling interval $\Delta$ such that $h_j = h(t_j)$. The adopted convention for the DFT is

$$\tilde{h}_k = \sum_{j=0}^{N-1} h_j e^{-i2\pi jk/N},$$

and the inverse DFT is then

$$h_j = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{h}_k e^{i2\pi jk/N},$$

where $i = \sqrt{-1}$.

2.2 Format for DFT’s in frequency series

In practice, the time domain data may be windowed before it is transformed. If the window function is described by $N$ real numbers $w_j$, then the windowed DFT is given by

$$\tilde{H}_k = \sum_{j=0}^{N-1} w_j h_j e^{-i2\pi jk/N},$$

where $i = \sqrt{-1}$. When defining the normalization for frequency series, we make reference to a quantity $\sigma_w$ which depends on the window function and is unity for a uniform window:

$$\sigma_w^2 = \frac{1}{N} \sum_{j=0}^{N-1} w_j^2.$$

The following conventions should be followed for frequency series:

- Meta-data consistent with the tabular description in Appendix A should be provided.
- The vector data[$p$] containing the frequency series data should be packed according to the following rules:
  1. data[0] should contain the lowest frequency component of the frequency series, i.e. the component at frequency $f_0$ which can be negative. For a complex time series with an even number $N$ of points, we adopt the convention that $\tilde{h}_{N/2}$ or $\tilde{H}_{N/2}$ is the lowest frequency component of the DFT.
  2. The frequency associated with the $p$th element of the vector is given by $f_0 + p \times df$. Thus, the frequency is always monotonically increasing with the index $p$.
- The DFT $\tilde{H}_k$ should be multiplied by the following normalization constant when packed into a frequency series:

$$\mathcal{A} = \frac{\Delta}{\sigma_w}.$$

Thus, elements of the vector data[$p$] will have units of seconds $\times$ (Units of time series). This normalization is chosen so that the elements in the frequency series will approximate a discretized version of (1).
Consider a concrete example involving the entire Fourier spectrum produced by a DFT of real data:
\[
data[k] = \frac{\Delta}{\sigma_w} \times \tilde{H}_k
\]  
(10)
for \(0 \leq k \leq [N/2]\) where \(\tilde{H}_k\) is defined in Eq. (7), \(\Delta\) is the sampling interval of \(h_j\), and \(\sigma_w\) is defined in Eq. (8). The notation \([x]\) means \(x\) rounded down to the nearest integer. The metadata \(f_0 = 0\) would indicate that \(k = 0\) corresponds to DC. Note that since negative frequencies Fourier components are determined by their positive frequency counterparts, they need not be included in the spectrum (see below).

- If the discrete time series \(\{h_j\}\) is real, the DFT will satisfy \(h_{N-k} = h_k^*\), and thus the elements with \(k > N/2\), which would represent negative frequencies, may be omitted from the frequency series. In this case, the symmetry metadatum should be set to 1, indicating that the absent negative frequency components may be inferred to be the complex conjugates of the corresponding positive frequency ones. If \(\{h_j\}\) represents the discretization of an inherently complex time series (not obtained by heterodyning a real time series), all \(N\) elements must be stored in the frequency series, and the symmetry metadatum set to 0 to indicate that there is no relationship between the positive and negative frequency components.

### 2.3 Fourier transforms of heterodyned time series

If \(\{h_j^h\}\) represents a discretization not of the original time-domain quantity \(h(t)\), but rather that quantity heterodyned and then filtered with an appropriate analog time-domain low-pass filter:
\[
h_j^h = (L * h)(t_0 + j \Delta)
\]  
(11)
and if the low-pass filter is assumed to ideally block frequencies above the Nyquist frequency corresponding to the sampling interval \(\Delta\), the DFT is related to the CFT of \(h(t)\) by
\[
\tilde{H}_k^h \sim \begin{cases} 
\frac{e^{-i\varphi_0}}{A} \tilde{h} \left( f_h + \frac{k}{N\Delta} \right) & k < \frac{N}{2} \\
\frac{e^{-i\varphi_0}}{A} \tilde{h} \left( f_h - \frac{N-k}{N\Delta} \right) & k \geq \frac{N}{2} 
\end{cases},
\]  
(12)
where \(A\) is the normalization constant defined in Section 2.2.

The following conventions should be followed when storing the DFT of a heterodyned function of time in a frequency series:

- Metadata consistent with Appendix A should be provided. In particular, the frequency metadata refer to the frequencies describing the underlying time-domain function, before heterodyning. For example, the physical start frequency \(f_0\) is given by \(f_h - \frac{N/2}{N\Delta}\).

- If the initial phase \(\varphi_0\) of the reference signal is known, it may be removed by setting
\[
data[k] = \begin{cases} 
e^{i\varphi_0} A \tilde{H}_k^h & k < [N/2] \\
e^{i\varphi_0} A \tilde{H}_k^h_{[N/2]} & k \geq [N/2] 
\end{cases}.
\]  
(13)
In that case, the symmetry metadatum should be set to 1, to indicate that we are dealing with a discrete representation of the CFT \(\tilde{h}(f)\) of a real function of time, and can infer its value at negative frequencies from positive frequency elements stored in the frequency series.
• If the initial phase \( \phi_0 \) is not removed (either by choice or because it is unknown), the elements in the frequency series will be

\[
data[k] = \begin{cases} A \tilde{H}_h^{k+N-[N/2]} & k < [N/2] \\ A \tilde{H}_h^{k-[N/2]} & k \geq [N/2] \end{cases},
\]

and the symmetry metadatum should be set to 2, to indicate the frequency series represents a constant unknown phase factor \( e^{-i\phi_0} \) times \( \tilde{h}(f) \) of a real function of time, and therefore can only infer the values at negative frequencies up to an overall phase.

### 3 Conventions for Power Spectral Densities [REQUIRED]

In this section, we discuss the normalization convention for the power spectral density (PSD) as used within the LSC. Estimation techniques for PSDs are not discussed, *per se*, although some examples are used to demonstrate the normalization explicitly.

For a vector of \( N \) time samples \( h_j \) with sampling interval \( \Delta \), the periodogram provides an estimate of the power distribution in the frequency domain. If \( k = 0 \) corresponds to DC, the periodogram is defined by

\[
P_0 = \frac{\Delta}{N} |\tilde{h}_0|^2
\]

\[
P_k = \frac{\Delta}{N} \{ |\tilde{h}_k|^2 + |\tilde{h}_{N-k}|^2 \}
\]

where \( \tilde{h}_k \) is given in Eq. (5) and \( 1 \leq k \leq [(N-1)/2] \). The notation \([x]\) means \( x \) rounded down to the nearest integer. When \( N \) is even, the Nyquist component is

\[
P_{N/2} = \frac{\Delta}{N} |\tilde{h}_{N/2}|^2.
\]

This definition respects Parseval’s Theorem in the form

\[
\Delta \sum_{j=0}^{N-1} |h_j|^2 = \sum_{k=0}^{[N/2]} P_k.
\]

The definition of the periodogram serves primarily to demonstrate the adopted normalization. It is only an estimate of the underlying power spectrum, and not a very good one at that.

A better estimate of the PSD can be obtained by averaging together the periodogram computed using several windowed DFTs. In this sense, a better estimate of the power spectrum is

\[
S_0 = \frac{\Delta}{N\sigma_w^2} \times \langle |\tilde{H}_0|^2 \rangle
\]

\[
S_k = \frac{\Delta}{N\sigma_w^2} \times \{ \langle |\tilde{H}_k|^2 \rangle + \langle |\tilde{H}_{N-k}|^2 \rangle \}
\]

where \( \tilde{H}_k \) is given in Eq. (7) and \( 1 \leq k \leq [(N-1)/2] \). The notation \([x]\) means \( x \) rounded down to the nearest integer. When \( N \) is even, the Nyquist component is

\[
S_{N/2} = \frac{\Delta}{N\sigma_w^2} \times \langle |\tilde{H}_{N/2}|^2 \rangle
\]

The notation \( \langle \ldots \rangle \) means average. Once again, Eqs. (19)-(21) serve to demonstrate the normalization convention for PSD’s and should not be taken to provide the correct low-level implementation.
3.1 Format for PSD’s in frequency series

Only one-sided PSD’s should be recorded as frequency series when the following conventions should be followed:

- Meta-data consistent with the tabular description in Appendix [Appendix A] should be provided.
- The vector $\text{PSD}[p]$ containing the frequency series data should be packed according to the following rules:
  1. $\text{PSD}[0]$ should contain the lowest frequency component of the frequency series, i.e. the component at frequency $f_0$.
  2. The frequency associated with the $p$th element of the vector is given by $f_0 + p \times df$. Thus, the frequency is always monotonically increasing with the index $p$.
- The vector containing a PSD will have units of seconds $\times$ (Units of time series)$^2$.

Consider a concrete example involving the full power spectrum (DC to Nyquist) for real data:

$$
\text{PSD}[0] = \frac{\Delta}{N \sigma_w^2} \times \langle |\tilde{H}_0|^2 \rangle \tag{22}
$$

$$
\text{PSD}[k] = \frac{\Delta}{N \sigma_w^2} \times \left\{ \langle |\tilde{H}_k|^2 \rangle + \langle |\tilde{H}_{N-k}|^2 \rangle \right\} \tag{23}
$$

where $\tilde{H}_k$ is defined in Eq. (7), $\Delta$ is the sampling interval of $h_j$, $\sigma_w$ is defined in Eq. (8) and $1 \leq k \leq \lfloor (N-1)/2 \rfloor$. When $N$ is even, the Nyquist component is

$$
\text{PSD}[N/2] = \frac{\Delta}{N \sigma_w^2} \times \langle |\tilde{H}_{N/2}|^2 \rangle. \tag{24}
$$

The notation $\lfloor x \rfloor$ means $x$ rounded down to the nearest integer. The metadata $f_0 = 0$ would indicate that $k = 0$ corresponds to DC. Equivalently, the one-sided PSD can be obtained from a frequency series containing all non-negative components of the DFT as

$$
\text{PSD}[0] = df \times \langle |\text{data}[0]|^2 \rangle \tag{25}
$$

$$
\text{PSD}[k] = 2 \times df \times \langle |\text{data}[k]|^2 \rangle \tag{26}
$$

where $\text{data}[k]$ is defined in Eq. (10) and $1 \leq k \leq \lfloor (N-1)/2 \rfloor$. When $N$ is even, the Nyquist component is

$$
\text{PSD}[N/2] = df \times \langle |\text{data}[N/2]|^2 \rangle. \tag{27}
$$

- A sample relationship between the PSD and the amplitude used to plot instrumental sensitivity is

$$
\text{amplitude}[k] = \sqrt{\text{PSD}[k]} \tag{28}
$$

where $0 \leq k \leq \lfloor N/2 \rfloor$, $\text{PSD}[k]$ is defined in Eqs. (22)–(24). The notation $\lfloor x \rfloor$ means $x$ rounded down to the nearest integer.
3.2 PSD conventions for heterodyned data

In the case of heterodyned data, as described in Section 2.3, \( k = 0 \) corresponds not to zero frequency, but to the heterodyning base frequency \( f_h \). Thus the definition of the one-sided PSD in this case will involve adding to the power at each positive frequency in the band the implied power at the corresponding negative frequency, which will be the same. (Note that this is true whether one has corrected for the initial phase or not, since that makes no difference to the amplitude.) Thus the elements of the PSD would be

\[
\text{PSD}[k] = \begin{cases} 
\frac{\Delta}{N \sigma_w^2} \times 2 \times \langle |\tilde{H}_{k+N-[N/2]}|^2 \rangle & k < [N/2] \\
\frac{\Delta}{N \sigma_w^2} \times 2 \times \langle |\tilde{H}_{k-[N/2]}|^2 \rangle & k \geq [N/2]
\end{cases}.
\]  

(29)

Equivalently, if \( \text{data}[k] \) represents a complex Fourier transform frequency series obtained from a heterodyned time series as described in Sec. 2.3 then

\[
\text{PSD}[k] = 2 \times df \times \langle |\text{data}[k]|^2 \rangle
\]  

(30)

where \( \text{data}[k] \) is defined in Eq. (13) or (14). As noted above, \( k = 0 \) corresponds to frequency \( f_0 = f_h \).
A Frequency Series Contents [REQUIRED]

Exchanged frequency series should have contents consistent with the following:

A.1 Mandatory Contents

These contents must be provided; there are no default values.

**Subtype** [4-byte integer or bit-field]
- 0 000 complex Fourier transform with linear frequency steps
- 1 001 complex Fourier transform with specified frequency values
- 2 010 real power spectrum with linear frequency steps
- 3 011 real power spectrum with specified frequency values
- 4 100 complex cross-spectrum with linear frequency steps
- 5 101 complex cross-spectrum with specified frequency values
- 6 110 real coherence with linear frequency steps
- 7 111 real coherence with specified frequency values

**Channel name** [Character string] The name of the channel (or pseudochannel), which should indicate the detector or site.

**Start time** [4-byte unsigned integer pair or 8-byte integer] GPS time of the start of the data.

**Stop time** [4-byte unsigned integer pair or 8-byte integer] GPS time of the end of the data.

**Number of points** [4-byte unsigned integer] Number of points of data in the frequency series.

**Frequencies**
For even-subtypes—those with a linear frequency steps—the following are required:

**Start frequency** [8-byte double precision real float] Physical start frequency in Hz.

**Frequency step** [8-byte double precision real float] Frequency spacing between elements in Hz.

For odd-subtypes—those with a specified frequency domain—the following are required:

**Frequency values** [Vector of 4-byte single precision real floats] The frequency, in Hz, of each frequency series value. The frequencies must be monotonically increasing.

**Series data** [Vector of 8-byte single precision complex floats or 4-byte single precision real floats] The values of the frequency series data. These correspond to monotonically-increasing frequencies.

A.2 Optional Contents

These contents may be omitted, in which case the default values are understood.

**Symmetry** [4-byte integer; default = 0 means no implied symmetry] The relation between positive frequency values and (unspecified) negative frequency values of the frequency series data:
- 0 No implied symmetry
- 1 Negative frequency values are complex-conjugates of positive frequency values
- 2 Negative frequency values are a constant (unknown) phase times the complex conjugate of positive frequency values
Units  [Character string; default = “” (empty string) means unspecified] The units associated with the values of the frequency series range data.

Power of ten  [4-byte signed integer; default = 0] Multiply the range data by \(10^{\text{power of ten}}\) to obtain normalization. (Needed to avoid underflow errors when storing strain power spectra.)

Window type  [4-byte integer; default = 0 is uniform window]

- 0   Uniform
- 1   Hanning
- 2   Flat-top
- 3   Welch
- 4   Bartlet
- 5   BMH
- 6   Hamming
- 7   Kaiser

FWHM spectral resolution  [8-byte double precision real float; default = 0 means unspecified] The full width at half maximum of the power spectrum of the window function.

Average type  [4-byte integer; default = 0 means unspecified]

- 0   Unspecified
- 1   Average of a fixed number of non-overlapping segment PSDs
- 2   Average of a fixed number of overlapping segment PSDs

Number of averages  [4-byte unsigned integer; default = 0 means unspecified] Number of data segments used in generating average power spectrum.

Exponential decay constant  [4-byte single precision real float; default = 0 means all segments are equally weighted] Exponential decay constant \(\alpha\) used in autoregressive moving average (“running average”) so the running average is weighted by a factor of \(e^{-\alpha}\) before including the present segment.

### A.3 Example C Structure [EXAMPLE]

Note: this is an illustrative example only—actual implementations may have a different structure.

```c
struct fseries {
    struct { unsigned isxyfmt : 1, ispower : 1, iscross : 1; } subtype;
    union { const char *name; const char *names[2]; } channel;
    struct { unsigned sec, nan; } tstart;
    struct { unsigned sec, nan; } tstop;
    unsigned npts;
    union { float *fval; struct { double f0, df; } step; } freq;
    union { struct { float re, im; } *cplx; float *real; } data;
    const char *units;
    int pow10;
    int symmetry;
    struct { int wintype; double resolution; } window;
    struct { int avgtype; unsigned navg; double decay; } average;
};
```